The Mediating Role of Strategic Competence Between Procedural Fluency and Conceptual Understanding in Mathematics

Sheila Joy O. Quilab[1] & Genelyn R. Baluyos[2]

[1,2] Misamis University, Ozamiz City, Philippines

DOI: https://doi.org/10.51244/IJRSI.2026.1305000112

Received: 07 May 2026; Accepted: 12 May 2026; Published: 03 June 2026

ABSTRACT

Mathematics learning becomes meaningful when students not only perform procedures accurately but also understand concepts and apply effective problem-solving strategies. This study examined the relationship between procedural fluency and conceptual understanding in mathematics learning, focusing on the mediating role of strategic competence among junior high school students in one of the basic education in Ozamiz City during the school year 2025–2026. A quantitative explanatory correlational design with mediation analysis was employed. The respondents consisted of 180 junior high school students, selected from a population of 337 through stratified random sampling, from a private non-sectarian educational institution in Ozamiz City, Misamis Occidental, Philippines. Data were gathered using three researcher-made instruments: the Procedural Fluency Questionnaire (PFQ), Conceptual Understanding Questionnaire (CUQ), and Strategic Competence Questionnaire (SCQ). Mean and standard deviation, Spearman's rank-order correlation, stepwise multiple regression analysis, and general linear mediation analysis were used to analyze the data. Results showed that students demonstrated high levels of procedural fluency, strategic competence, and conceptual understanding. Significant positive relationships existed among students' procedural fluency, strategic competence, and conceptual understanding. Efficiency in problem solving, flexibility in applying procedures, appropriateness of procedures, problem representation, strategy formulation, evaluation, and justification were predictive of conceptual understanding. Strategic competence significantly mediated the relationship between procedural fluency and conceptual understanding, indicating a partial mediation effect. The findings highlight that students demonstrated high levels of procedural fluency, strategic competence, and conceptual understanding in mathematics, with significant positive relationships among these variables, where problem-solving efficiency and related skills predicted conceptual understanding, and strategic competence partially mediated the link between procedural fluency and conceptual understanding. Mathematics instructors may promote the balanced development of procedural accuracy, conceptual reasoning, and strategic problem-solving to support students' holistic mathematical proficiency.

Keywords: conceptual understanding, mathematics learning, mediation analysis, procedural fluency, strategic competence

INTRODUCTION

Mathematics education is increasingly viewed as a process that extends beyond mastering computational procedures towards the development of meaningful understanding and problem-solving ability. Research has consistently emphasized that effective mathematics learning requires integrating interrelated competencies, particularly procedural fluency, conceptual understanding, and strategic competence (Diputra et al., 2025). However, despite curriculum revisions and pedagogical advances, many learners continue to exhibit fragmented mathematical proficiency, defined by procedural accuracy without deep conceptual meaning or flexible problem-solving skills (Corrêa, 2021). Educators and researchers have questioned the effectiveness of current instructional practices in promoting holistic mathematical learning due to this persistent imbalance.

Procedural fluency refers to students' ability to carry out mathematical procedures accurately, efficiently, and flexibly. Research findings indicate that many students demonstrate acceptable procedural performance, particularly on familiar tasks and structured exercises (Andal, 2022). However, several studies reveal that procedural fluency is frequently developed by rote learning and repetitive practice, leaving students unable to rationalize solutions or adapt procedures to new situations (Annurwanda, 2022). Further evidence indicates that students often possess more procedural than conceptual knowledge, suggesting that procedural fluency alone does not ensure meaningful understanding (Manandhar, 2022).

Conceptual understanding involves comprehending mathematical concepts, relationships, and representations, allowing learners to explain ideas, make connections, and apply knowledge across contexts. Research consistently shows that students with a solid foundation of conceptual understanding perform better on problem-solving and reasoning tasks (Kholid, 2021). However, research also shows that conceptual understanding is often lacking among students, particularly in algebraic and abstract topics, due to instructional approaches that prioritize procedural accuracy over conceptual meaning (Copur-Gencturk, 2021). Research further confirms a strong relationship between procedural fluency and conceptual understanding, yet highlights that this relationship is not automatic and requires sufficient cognitive and instructional support (Cabuquin, 2024).

Strategic competence has emerged as a crucial factor in addressing this gap, as it involves the ability to formulate, represent, and solve mathematical problems effectively. Strategic competence enables students to select appropriate procedures, adapt strategies, and translate real-world situations into mathematical representations (Schulz, 2024). Research findings indicate that students who demonstrate strong strategic competence exhibit greater flexibility in strategy application and improved problem-solving performance (Verschaffel, 2024).

In this context, the concept of a mediating role is essential for understanding how these variables interact. A mediating variable is a variable that explains the mechanism or process by which one variable influences another. In this study, strategic competence is hypothesized to mediate the relationship between procedural fluency and conceptual understanding, meaning that procedural fluency may not directly lead to conceptual understanding unless it is supported by students' ability to use strategies effectively (Supriadi & Suherman, 2024). This perspective shifts the focus from merely identifying relationships to explaining how and why they occur, providing a deeper understanding of the learning process.

Moreover, studies on mathematical proficiency reveal that students with stronger strategic competence are better able to integrate procedural knowledge with conceptual understanding, thereby justifying their solutions and drawing meaningful connections among mathematical ideas (Mohamed Elsayed, 2022). Empirical evidence suggests that strategic competence plays a significant role in distinguishing high-achieving students from their peers. Research on primary and secondary learners revealed that strategy variety and flexibility are major determinants of mathematical achievement (Bakker, 2022). Similarly, assessments of prospective teachers revealed that difficulties in mathematical performance were often linked to deficiencies in integrating procedures with strategic reasoning rather than to procedural errors alone (Elvi, 2025). These findings suggest that strategic competence serves as a cognitive bridge, transforming procedural execution into conceptual insight.

Although numerous studies have examined procedural fluency, conceptual understanding, and strategic competence as components of mathematical proficiency, most existing research has focused either on their direct relationships or on treating them as parallel entities. Limited empirical studies have directly examined how strategic competence functions as a mediating variable between procedural fluency and conceptual understanding (Corrêa, 2021). Recent findings acknowledge the relevance of strategic competence, but its role in explaining how procedural skills translate into conceptual understanding remains unclear (Khaerunnisa, 2025). This gap underscores the need for a quantitative mediation study examining the indirect effect of procedural fluency on conceptual understanding via strategic competence, particularly in secondary mathematics contexts where conceptual difficulties are most evident.

This study is grounded in the need to address the persistent imbalance between procedural performance and conceptual understanding in mathematics learning. Research findings consistently reveal that students' difficulties in problem representation, strategy selection, and solution evaluation hinder their ability to develop deep conceptual understanding, even when procedural skills are present (Hariri, 2025). By examining strategic competence as a mediating variable, this study seeks to explain how procedural fluency contributes to conceptual understanding. Understanding this mechanism is essential for designing instructional practices that go beyond memorization and promote meaningful mathematical learning.

The purpose of this study is to determine whether strategic competence mediates the relationship between

procedural fluency and conceptual understanding among mathematics students. The study aims to assess students' procedural fluency through routine and adaptive computational tasks, assess conceptual understanding through problem-solving and explanation tasks, and evaluate strategic competence through tasks that require strategy selection, planning, and the flexible application of procedures. A correlational-mediation research design is employed to explore the interconnections among the three variables in authentic learning contexts.

This study is significant because it provides educators with a deeper understanding of how procedural fluency and conceptual understanding interact through strategic competence. Insights from this research can inform teaching practices, curriculum design, and assessment strategies that foster a more balanced development of mathematical proficiency. Additionally, findings may guide teacher training programs to emphasize the cultivation of strategic competence alongside procedural and conceptual skills.

Statement of the Problem

This study aimed to explore procedural fluency and conceptual understanding in mathematics learning, with a focus on the mediating role of strategic competence. The study was conducted in junior high education during the school year 2025-2026.

Specifically, it sought answers to the following questions:

1. What is the level of students' procedural fluency in mathematics learning in terms of accuracy, flexibility, efficiency, and appropriateness?
2. What is the level of students' strategic competence in mathematics learning in terms of problem representation, strategy formulation, adaptation of methods, and evaluation and justification of solutions?
3. What is the level of students' conceptual understanding in mathematics learning in terms of comprehension of principles, application of concepts, connections across topics, and reasoning and justification?
4. Is there a significant relationship between students' procedural fluency and their strategic competence?
5. Is there a significant relationship between students' strategic competence and their conceptual understanding?
6. Is there a significant relationship between students' procedural fluency and conceptual understanding in mathematics learning?
7. Which aspects of the variables significantly predict students' conceptual understanding of mathematics learning?
8. What is the mediating effect of strategic competence on the relationship between procedural fluency and conceptual understanding in mathematics learning?

RESEARCH METHODOLOGY

Design

This quantitative study employed an explanatory correlational design with mediation analysis to examine the relationships among procedural fluency, conceptual understanding, and strategic competence in mathematics learning. Correlational research designs were used to describe and explain relationships among variables using naturally occurring data without manipulation (Creswell, 2012). This design was appropriate as it enables the researcher to determine the degree and direction of relationships among the identified variables and to examine how they interact within an actual educational setting. The inclusion of strategic competence was supported by the literature, which indicated that the presence of a third variable provided a clearer, more elaborated understanding of the relationship between an independent and a dependent variable by explaining how the association operated (Sidhu et al., 2021). The design was appropriate for the present study because it focuses on identifying significant relationships among the variables and clarifying the role of strategic competence within the relationship between procedural fluency and conceptual understanding in mathematics.

Setting

The study took place in a private, non-sectarian higher education institution located in Ozamiz City, Misamis Occidental, Philippines. In 2024, the institution earned ISO 21001:2018 certification and was recognized as the only Autonomous Higher Education Institution in Misamis Occidental and Northwestern Mindanao. It was widely acknowledged for its commitment to academic excellence, inclusivity, and continuous quality enhancement. With its strong adoption of educational technologies in mathematics instruction and its focus on holistic student development, the institution provided an appropriate setting to investigate the roles of procedural fluency and conceptual understanding, as well as the mediating influence of strategic competence in mathematics learning.

Respondents

The respondents in this study were 180 students from a junior high school. They were selected from a total population of 337 junior high school students enrolled in a private, non-sectarian institution during the school year 2025–2026. The sample size of 180 respondents was determined using the Raosoft sample size calculator, with a 95% confidence level and a 5% margin of error. To ensure adequate representation across grade levels, stratified random sampling was employed, grouping the population by grade level. The inclusion criteria are as follows: (1) students were officially enrolled during the specified school year; (2) they were enrolled in Junior High School; and (3) they provided voluntary assent with corresponding parental or guardian consent. Prior to data collection, the researcher verified that all selected respondents met these criteria to ensure the validity and reliability of the study.

Instruments

Procedural Fluency Questionnaire (PFQ). The researcher-made instrument was designed to assess procedural fluency in Mathematics among junior high school students. It was anchored on the DepEd MATATAG Mathematics Curriculum Guide for Key Stage 3. It measured procedural fluency in terms of accuracy, efficiency, flexibility, and appropriateness of procedures using a five-point Likert scale ranging from 1 (Very High) to 5 (Very Low). The instrument was intended for junior high school students and aligned with competencies taught during the fourth grading period. Prior to administration, the questionnaire underwent content and face validation by Mathematics education experts to ensure curricular conformity, clarity, and relevance. Pilot testing was then conducted, yielding a Cronbach's Alpha of 0.939, confirming the instrument's high reliability for this study.

In assessing the level of procedural fluency, the following scale was used:

Responses	Continuum	Interpretation
5 – Strongly Agree (SA)	4.20 – 5.00	Very High (VH)
4 – Agree (A)	3.40 – 4.19	High (H)
3 –Neutral (N)	2.60 – 3.39	Moderate (M)
2 –Disagree (D)	1.80 – 2.59	Low (L)
1 –Strongly Disagree (SD)	1.0 – 1.79	Very Low (VL)

The researcher-made instrument was designed to assess learners' level of strategic competence in mathematics and was anchored on the DepEd MATATAG Mathematics Curriculum Guide for Key Stage 3. It measured strategic competence in terms of the problem representation, strategy formulation, adaptation and flexibility, and evaluation and justifications, using a five-point Likert scale ranging from 1 (Very Low) to 5 (Very High). The instrument was intended for junior high school students and aligned with competencies taught during the fourth grading period. Prior to administration, the questionnaire will undergo content and face validation by Mathematics education experts to ensure curricular conformity, clarity, and relevance. Pilot testing was then conducted, yielding a Cronbach's Alpha of 0.941, confirming the instrument's high reliability for this study.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue V May 2026

In assessing the level of strategic competence, the following scale was used:

Responses	Continuum	Interpretation
5 – Strongly Agree (SA)	4.20 – 5.00	Very High (VH)
4 – Agree (A)	3.40 – 4.19	High (H)
3 –Neutral (N)	2.60 – 3.39	Moderate (M)
2 –Disagree (D)	1.80 – 2.59	Low (L)
1 –Strongly Disagree (SD)	1.0 – 1.79	Very Low (VL)

Conceptual Understanding Questionnaire (CUQ). The researcher-made instrument was designed to assess the level of procedural fluency in Mathematics among junior high school students. It was anchored on the DepEd MATATAG Mathematics Curriculum Guide for Key Stage 3. It measured procedural fluency in terms of accuracy, efficiency, flexibility, and appropriateness of procedures using a five-point Likert scale ranging from 1 (Very High) to 5 (Very Low). The instrument was intended for junior high school students and aligned with competencies taught during the fourth grading period. Prior to administration, the questionnaire underwent content and face validation by Mathematics education experts to ensure curricular conformity, clarity, and relevance. Pilot testing was then conducted, yielding a Cronbach's Alpha of 0.953, confirming the instrument's high reliability for this study.

In evaluating the level of conceptual understanding, the following scale was used:

Responses	Continuum	Interpretation
5 – Strongly Agree (SA)	4.20 – 5.00	Very High (VH)
4 – Agree (A)	3.40 – 4.19	High (H)
3 –Neutral (N)	2.60 – 3.39	Moderate (M)
2 –Disagree (D)	1.80 – 2.59	Low (L)
1 –Strongly Disagree (SD)	1.0 – 1.79	Very Low (VL)

Data Gathering Procedure

Prior to data collection, the researcher sought approval from the Dean of the Graduate School, followed by authorization from the Vice President for Academic Affairs, a formal letter from the Student Affairs Director, and permission from the Principal of the Basic Education department to conduct the study within the school premises. After securing all approvals, potential respondents, including both mathematics teachers and students, were screened for eligibility, and informed consent was obtained to ensure voluntary participation. Data were collected personally by the researcher through the distribution of questionnaires, which were retrieved immediately after completion. The gathered data were organized, encoded, and subjected to statistical analysis, followed by interpretation of the findings.

Ethical Consideration

This study obtained approval from the Misamis University Research Ethics Committee (MUREC) prior to its commencement. The well-being of respondents was prioritized, ensuring their safety, dignity, and comfort throughout the study. Since the respondents were minors, both assent forms and informed consent forms were secured before data collection. The assent form was given to the respondents, while written consent was obtained from their parents or legal guardians. These documents clearly explained the study's purpose, procedures, potential risks, and benefits. Participation was strictly voluntary, and respondents could withdraw at any time without any negative consequences. To avoid disruption of academic activities, the survey was conducted beyond regular class hours at a time convenient for the respondents.

Confidentiality was strictly maintained by anonymizing respondents' identities and by securely storing all collected data in a location accessible only to the researcher. The data were retained for a specified period, such as one year after the study's completion, and were disposed of properly thereafter. The study remained transparent in its objectives and procedures, with no form of deception involved. While there are no monetary incentives or payments for participation, respondents could benefit from increased awareness and understanding of the research topic. All ethical guidelines were strictly followed to ensure the integrity and credibility of the study.

Data Analysis

The study used the following statistical tools in analyzing the data gathered:

Mean and Standard Deviation. These tools were used to assess students' procedural fluency, conceptual understanding, and strategic competence in mathematics.

Spearman's Rank-Order Correlation. This tool was used to explore the significant relationships among procedural fluency and conceptual understanding, procedural fluency and strategic competence, and strategic competence and conceptual understanding in mathematics learning.

Stepwise Multiple Regression Analysis. This tool was utilized to identify which aspect of procedural competence significantly predicts conceptual understanding in mathematics learning.

General Linear Mediation Analysis. This tool was used to test the mediating effect of strategic competence on the relationship between procedural fluency and conceptual understanding in mathematics learning.

RESULTS AND DISCUSSION

Level of Students' Procedural Fluency

Table 1 presents students' procedural fluency in mathematics, indicating overall high performance (M = 3.60, SD = 0.70). This indicates that students generally demonstrate competence in executing mathematical procedures, reflecting their ability to calculate and solve problems effectively. The relatively small standard deviation suggests moderately consistent performance among students, indicating a shared baseline in procedural skills. While foundational abilities are strong, improvement is needed in applying procedures to more complex or unfamiliar problem contexts.

Procedural fluency demonstrates students' ability to accurately carry out mathematical procedures, complete solutions in a reasonable time, and adapt or select appropriate strategies when faced with different kinds of problems. Strong procedural fluency enables students to follow algorithms step-by-step, apply them meaningfully, and adapt their approaches to the job at hand. As a result, evaluating procedural fluency enables teachers to pinpoint students' strengths and weaknesses in performing mathematical operations, which can help them create targeted instructional strategies that improve students' overall mathematical proficiency and enhance learning outcomes.

Students' accuracy in solving mathematical problems was high (M = 3.74, SD = 0.73). This suggests that students typically complete tasks accurately, with few mistakes. This degree of accuracy appears consistent across students, as indicated by the comparatively low standard deviation. Students can perform computations with confidence and accuracy because they have a strong grasp of basic procedures. They can reduce errors and ensure accuracy in their solutions by performing mathematical calculations with precision. This degree of precision also facilitates more efficient problem-solving, since accurate initial stages provide a solid basis for subsequent processes. This finding suggests that students have strong procedural skills, which greatly enhance their overall proficiency in mathematics.

Students' procedural appropriateness was practiced at a high level (M = 3.64, SD = 0.83). This suggests that students can generally accomplish mathematical tasks promptly without sacrificing accuracy. However, the significantly higher standard deviation indicates performance variability, implying that some students can answer problems more rapidly than others, requiring more time and guidance. Efficient students can balance speed and precision, making it easier for them to complete multi-step mathematical tasks. Through this ability, they can effectively manage their time while preserving the quality of their solutions. This finding implies that, even though many students are proficient at solving problems, there is still a need to assist individuals who need help controlling both speed and accuracy in mathematical tasks.

Students' problem-solving efficiency was high (M = 3.54, SD = 1.03). This suggests that students can generally accomplish mathematical tasks promptly without sacrificing accuracy. However, the significantly higher standard deviation indicates performance variability, implying that some students can answer problems more rapidly than others, requiring more time and guidance. Efficient students can balance speed and precision, making it easier for them to complete multi-step mathematical tasks. Through this ability, they can effectively manage their time while preserving the quality of their solutions. This finding implies that, even though many students are proficient at solving problems, there is still a need to assist individuals who need help controlling both speed and accuracy in mathematical tasks.

Students' flexibility in applying procedures was moderate (M = 3.49, SD = 0.82). This indicates that while students are capable of using multiple strategies, adapting procedures to unfamiliar or novel problems may still be challenging. The moderate standard deviation suggests variability in students' ability to transfer or modify procedural knowledge, with some students demonstrating greater adaptability than others. Developing flexibility is essential for connecting procedural skills to effective problem-solving and for fostering innovation, as it enables students to respond confidently to new and complex mathematical challenges. This result highlights the need to further support students in applying procedures flexibly across different problem contexts.

Procedural fluency is widely recognized by experts as a critical component of mathematical performance, particularly for predicting accuracy and efficiency levels (Vanderheyden & Solomon, 2023). Higher fluency results in more suitable and effective approaches, which align with the high scores found (Skylark, 2021). The slightly lower mean for this construct is consistent with the fact that flexibility develops more slowly than accuracy and efficiency (Hong et al., 2023). Procedural skills are the foundation for conceptual understanding (Hurrel, 2021), and secondary learners demonstrated strong overall fluency, supporting these findings (Landreth & Young, 2021).

Students possess strong procedural skills overall, particularly in accuracy and appropriateness, though their flexibility is significantly lower. Therefore, the focus of instruction should be on helping students become more adept at adapting processes to new or challenging circumstances. This can be accomplished through tasks that call for a variety of problem-solving approaches, problem variation, and reflective exercises that push students to think beyond standard procedures. In addition to improving adaptive problem-solving, strengthening flexibility ensures that students can effectively apply their knowledge across a variety of circumstances by bridging procedural knowledge with deeper conceptual understanding.

Table 1 Level of Students' Procedural Fluency

Constructs	M	SD	Interpretation
Accuracy in Solving Procedures	3.74	0.73	High
Efficiency in Problem Solving	3.54	1.03	High
Flexibility in Applying Procedures	3.49	0.82	High
Appropriateness of Procedures	3.64	0.83	High
Overall Level of Students' Procedural Fluency	3.60	0.70	High

Scale: 4.20-5.00(Very High); 3.40-4.19 (High); 2.60-3.39 (Moderate); 1.80-2.59 (Low);1.0-1.79 (Very Low)

Level of the Students' Strategic Competence

Table 2 presents the level of students' strategic competence in mathematics, indicating overall high performance (M = 3.65, SD = 0.69). This indicates that students generally demonstrate strong competence in using strategies that support effective problem-solving. The relatively small standard deviation suggests moderately consistent performance among students, indicating a shared proficiency in strategic skills. While the overall level is high, continued development of strategic processes, such as problem representation and solution evaluation, can further enrich students' mathematical reasoning and decision-making.

Strategic competence highlights students' ability to comprehend the structure of a problem, convert it into mathematical form, and devise relevant tactics to achieve an effective solution. When challenges arise, students who exhibit strong strategic competence can adjust their strategies, choose multiple approaches, and evaluate whether their solutions are reasonable and logically supported. As a result, assessing strategic competence provides teachers with important information on how well students combine procedural knowledge and conceptual understanding, enabling them to foster problem-solving abilities that support the development of holistic mathematical ability.

The adaptation and flexibility was interpreted as being practiced to a very high extent (M = 3.70, SD = 0.81). This implies that when their initial plans are inadequate, students are highly capable of modifying techniques or approaches. This adaptability shows a high degree of metacognitive awareness, as students monitor how they solve problems and adjust as needed. When students adjust well, they show resilience and self-assurance in managing challenging tasks, which promotes success across a variety of mathematical problems. This flexibility improves students' general problem-solving skills and equips them to react strategically to new or unforeseen circumstances.

Likewise, evaluation and justification was interpreted as being practiced to a very high extent (M = 3.69, SD = 0.88). This shows that students can evaluate the precision of their answers and justify the strategies they select. This ability is essential for verifying the validity of results, reflecting on problem-solving procedures, and strengthening deeper mathematical understanding, even though students differ in how they express or defend their reasoning. Students with strong evaluation and justification skills are better able to make deliberate choices and enhance the quality of their mathematical reasoning.

Strategy formulation was interpreted as being practiced to a very high extent (M = 3.63, SD = 0.82). This result demonstrates how proficient students are in devising strategies and solutions to given problems. To solve problems effectively and successfully across a variety of mathematical contexts, students must be able to create plans tailored to the specific demands of each situation. This skill helps students approach challenges systematically and boosts their confidence in applying appropriate strategies.

Problem representation was interpreted as being practiced to a very high extent (M = 3.58, SD = 0.86). This suggests that students are generally competent at interpreting and organizing information in mathematical tasks. Some students may need assistance in transforming problems into mathematical formats or visual representations, even though most students can accurately represent them. Competence in problem representation serves as a basis for systematic problem-solving and supports students in selecting and using appropriate strategies.

Contemporary research underscores the importance of strategic competence in mathematics. For instance, strong problem representation abilities allow students to better frame problems, resulting in clearer solution pathways (Dood & Watts, 2022). Explicit strategy formulation activities have been found to improve students' performance in a variety of problem scenarios (Teng, 2022). Research emphasizes that flexibility in strategy use is closely linked to metacognitive regulation, enabling learners to adjust their approaches when initial plans fail (Bürgler et al., 2021). Additionally, students' mathematical argumentation and reasoning are improved by evaluation and justification skills, which promote deeper learning and reflective competency (Santosa et al., 2025). Collectively, these studies affirm that strategic processes are central to mathematical success and align with the high levels of strategic competence observed in this study.

Students demonstrate high strategic competence across all areas, with flexibility and adaptation slightly ahead. Instructional strategies should emphasize problem representation, strategy formulation, and evaluation skills while retaining flexibility to continue to build on this strength. Students' strategic decision-making can be further improved by assigning them demanding, real-world problem-solving tasks that call for strategy development, monitoring, and adjustment. Strengthening these areas promotes independent, flexible, and reflective mathematical thinking by ensuring that students not only carry out efficient problem-solving procedures but also critically assess and defend their strategies.

Table 2 Level of Students' Strategic Competence

Constructs	M	SD	Interpretation
Problem Representation	3.58	0.86	High
Strategy Formulation	3.63	0.82	High
Adaptation and Flexibility	3.70	0.81	High
Evaluation and Justification	3.69	0.88	High
Overall Level of Competence	3.65	0.69	High

Scale: 4.20-5.00(Very High); 3.40-4.19 (High); 2.60-3.39 (Moderate); 1.80-2.59 (Low);1.0-1.79 (Very Low)

Level of the Students' Conceptual Understanding

Table 3 presents the level of students' conceptual understanding in mathematics, indicating overall high performance (M= 3.47, SD = 0.75). This indicates that students generally demonstrate competence in understanding and applying mathematical concepts, reflecting their ability to reason, make connections, and solve problems effectively. The relatively small standard deviation suggests moderately consistent performance among students, indicating a shared baseline in conceptual skills. While students demonstrate a high level of understanding, there is room for improvement in higher-order reasoning and justification, particularly when linking concepts across different problem contexts.

Conceptual understanding emphasizes students' capacity to comprehend the meaning underlying mathematical concepts rather than simply memorizing processes or formulas. Strong conceptual understanding enables students to apply their knowledge to new or real-world scenarios, explain why specific strategies work, and analyze mathematical relationships. As a result, evaluating conceptual understanding enables educators to gauge the depth of students' mathematical thinking and to develop instructional strategies that strengthen reasoning abilities, foster meaningful connections between concepts, and promote deeper, longer-lasting learning.

The application of problem-solving concepts was interpreted as practiced to a high extent (M = 3.53, SD = 0.81). This result suggests that students can typically use the mathematical concepts they have learned to solve problems effectively. This level of application allows students to integrate their conceptual understanding into practical solutions and engage in meaningful problem-solving tasks. They can relate mathematical concepts to real-world circumstances and procedural processes, helping them to approach challenges with greater confidence and flexibility. As a result, students have improved analytical abilities and are better equipped to handle a variety of mathematical challenges. This ability to put ideas into practice promotes deeper learning and strengthens the integration of mathematical knowledge and skills.

Closely following this, comprehension of mathematical concepts was interpreted as being practiced to a high extent (M = 3.52, SD = 0.80). This result suggests that students typically exhibit a solid understanding of basic mathematical concepts and ideas. Students can recognize, explain, and comprehend essential concepts, which enhances their capacity to engage in meaningful mathematical reasoning. This level of understanding enables students to connect new and existing knowledge, thereby developing a deeper understanding of mathematical relationships. Students are therefore better equipped to apply their knowledge to problem-solving and tackle increasingly difficult mathematics tasks. This level of conceptual understanding fosters higher-order mathematical thinking and provides a solid foundation for continuous learning.

Making connections across topics was interpreted as being practiced to a high extent (M = 3.47, SD = 0.85). This result suggests that students can generally relate mathematical concepts across and between topics, while this skill may be difficult for some individuals. Through this approach, students begin to recognize connections among mathematical ideas, helping them develop a more structured, cohesive grasp of the subject. Deeper understanding and retention are supported by their ability to connect previously taught material with new material. As a result, students progressively gain a more comprehensive understanding of mathematics, improving their capacity for critical thought and problem-solving. Developing this ability is crucial for encouraging comprehensive mathematical reasoning and creating a more fulfilling educational experience.

Lastly, reasoning and justification was likewise interpreted as being practiced to a moderate extent (M = 3.36, SD = 0.95). This result suggests that students have greater difficulties expressing their problem-solving steps, presenting clear justifications, and creating logical mathematical arguments. This makes it clear that many students lack the confidence to articulate the reasoning behind their answers, which could limit the depth of their understanding. As a result, students may rely more on steps than on solid explanations of mathematical concepts. Because it allows students to relate their information to logical explanations, strengthening reasoning and justification is crucial for promoting deeper conceptual understanding. Additionally, this ability is crucial for fostering higher-order thinking and bridging the gap between procedural fluency and deep mathematical understanding.

Research indicates that conceptual knowledge is crucial for effective learning in mathematics, as it enables pupils to apply concepts across a variety of situations and problems (Naz & Qayyum, 2025). Stronger conceptual skills improve students' problem-solving performance and engagement, supporting the high levels of comprehension and application observed in this study (Siregar, 2025). Concept-based instructional approaches improve learners' knowledge of key mathematical concepts, as evidenced by observable comprehension and application (Ncube and Luneta, 2025).

Students' ability to shift between representations improves reasoning and connections across topics, which relates to the moderate but growing scores in making connections (Haque, 2024). Furthermore, context-based instruction encourages reasoning and justification, emphasizing the need to focus on these skills (Nurtamam & Jannah, 2025), which recorded the lowest mean in this study. Collectively, these results highlight the significance of targeted instructional strategies to strengthening reasoning and justification skills while preserving high levels of understanding, application, and conceptual connections.

Although the overall level of students' conceptual understanding is interpreted as high, the result for reasoning and justification falls within the moderate range, suggesting a relative weakness. Instruction should focus on activities that require students to articulate their thought processes, justify solutions, and make connections across topics. Integrating reflective discussions, problem-based tasks, and cross-topic exercises can help students deepen their reasoning skills while maintaining strengths in comprehension, application, and conceptual connections. Enhancing reasoning and justification fosters critical thinking and ensures that students can not only solve problems but also explain, analyze, and generalize mathematical concepts.

Table 3 Level of Students' Conceptual Understanding

Constructs	M	SD	Interpretation
Comprehension of Mathematical Concepts	3.52	0.80	High
Application of Concepts in Problem-Solving	3.53	0.81	High
Making Connections Across Topics	3.47	0.85	High
Reasoning and Justification	3.36	0.95	Moderate
Overall Level of Understanding	3.47	0.75	High

Scale: 4.20-5.00(Very High); 3.40-4.19 (High); 2.60-3.39 (Moderate); 1.80-2.59 (Low); 1.0-1.79 (Very Low)

Significant Relationship between Students' Procedural Fluency and Strategic Competence

Table 4 presents the relationships between students' procedural fluency and strategic competence in mathematics, revealing strong, highly significant correlations across all constructs. Spearman's rho values range from 0.473 to 0.644 ($p < .001$), indicating that students who demonstrate higher accuracy, efficiency, flexibility, and appropriateness in procedural execution also exhibit higher levels of problem representation, strategy formulation, adaptation, and flexibility, and evaluation and justification. The data highlight that procedural fluency forms a foundational platform for strategic thinking, enabling learners to plan, adapt, and evaluate their mathematical approaches effectively.

Procedural fluency serves as a cognitive foundation that enables strategic engagement. Students who can perform procedures accurately, efficiently, and adaptively can devote cognitive resources to analyzing problem structures, planning approaches, and monitoring the effectiveness of their solutions. This relationship suggests that mastery of procedural abilities reduces cognitive burden, allowing pupils to focus on higher-order strategic processes rather than simple computations. Procedural fluency, therefore, acts as a scaffold for developing strategic competence, supporting both efficiency and adaptability in problem-solving.

Accuracy in solving procedures shows strong, highly significant correlations with problem representation (r_s = 0.529), strategy formulation (r_s = 0.474), adaptation and flexibility (r_s = 0.571), and evaluation and justification (r_s = 0.477), all $p < .001$. The strongest relationship is with adaptation and flexibility, indicating that students who follow procedures correctly are better able to adjust techniques when handling unfamiliar problems. Accurate procedural execution helps students understand problem structures more clearly, supports effective strategy selection, and enables them to evaluate and justify their solutions with greater confidence.

Efficiency in problem solving also demonstrates strong and highly significant correlations with problem representation (r_s = 0.525), strategy formulation (r_s = 0.473), adaptation and flexibility (r_s = 0.575), and evaluation and justification (r_s = 0.563), all with $p < .001$. The strongest relationships are observed with adaptation and flexibility, as well as with evaluation and justification, suggesting that students who solve problems effectively can better monitor and adjust their strategies. Effective problem solvers focus less on fundamental procedural steps and more on evaluating the effectiveness of their strategies.

Flexibility in applying procedures shows strong correlations with problem representation (r_s = 0.526), strategy formulation (r_s = 0.504), adaptation and flexibility (r_s = 0.567), and evaluation and justification (r_s = 0.555), all with $p < .001$. These results indicate that students who can modify procedures for different problem situations are better able to develop appropriate strategies and assess the effectiveness of their solutions. Procedural flexibility, therefore, supports adaptive thinking and strategic problem solving.

Appropriateness of procedures exhibits the strongest relationships, particularly with adaptation and flexibility (r_s = 0.644) and evaluation and justification (r_s = 0.583), while maintaining strong correlations with problem representation (r_s = 0.529) and strategy formulation (r_s = 0.473), all with $p < .001$. This suggests that selecting suitable procedures helps students adjust their strategies effectively and justify their solutions, highlighting the importance of procedural decision-making in strengthening strategic competence.

Several studies support these findings. Procedural fluency supports students' ability to accurately represent problems and develop successful strategies (Andal, 2022). Efficiency and accuracy in procedural execution predict students' adaptive problem-solving and evaluation skills (Brancaccio et al., 2023). Flexibility in procedural strategies facilitates strategy formulation and critical assessment (Star et al., 2022). Students who select appropriate procedures demonstrate stronger reasoning and justification (Öz & Işık, 2024). Integrating procedural mastery with strategic practice enhances problem-solving independence (Singh et al., 2023). Students with well-developed procedural skills can monitor and refine strategies more effectively (Vettriselvan et al., 2025), while procedural fluency is essential for bridging execution with strategic thinking (Applebaum, 2024). Collectively, these studies corroborate the highly significant correlations observed in this study.

The Information Processing Theory (IPT) suggests that students who can efficiently encode, store, and retrieve mathematical procedures develop automaticity in routine calculations (Pratt, 2023). This automaticity reduces cognitive load, allowing students to allocate more mental resources to higher-order thinking processes such as strategy formulation, problem representation, and evaluation (Yassin, 2024). As a result, learners are better able to engage in complex mathematical tasks because they no longer need to devote excessive attention to basic computational steps. In this way, procedural fluency supports the development of more advanced cognitive processes involved in mathematical problem solving.

Complementing this perspective, the Mathematical Proficiency Framework emphasizes that strategic competence, including problem representation, strategy formulation, adaptation, and evaluation, is essential for flexible, reflective, and effective problem solving (Kilpatrick, Swafford, & Findell, 2001). Procedural fluency and strategic competence, therefore, serve as mutually reinforcing components of mathematical proficiency. Reliable procedural skills provide the mental bandwidth required to plan, monitor, and adjust problem-solving approaches, whereas active strategic engagement increases the application and transfer of procedural knowledge (Karatas, 2022). Empirical studies reinforce this relationship, indicating that students with procedural automaticity are more capable of adaptive problem solving and evaluating alternative strategies (Hickendorff, 2022), that strong procedural foundations allow thinking and reflective decision-making (Dingman et al., 2021), and that procedural fluency predicts students' capacity to generate strategies and adapt approaches in unfamiliar contexts (Schulz, 2024). Collectively, these findings indicate that procedural mastery acts as a cognitive scaffold for the development of strategic competence and overall mathematical proficiency.

Improving procedural fluency can significantly strengthen students' strategic competence. Since adaptation, flexibility, and evaluation show the strongest relationships with procedural fluency, instructional strategies should emphasize tasks that require students to select appropriate methods, alter strategies for novel problems, and reflect critically on their solutions. Problem modeling, reflective discussion, and strategy journaling are all activities that can help students improve adaptive thinking, strategy formulation, and evaluation skills. Enhancing procedural flexibility and appropriateness not only promotes proper execution but also fosters independent, efficient, and comprehensive strategic problem-solving across a wide range of mathematical contexts.

Table 4 Significant Relationship between Students' Procedural Fluency and Strategic Competence

Variables		Problem Representation	Strategy Formulation Efficiency in Problem Solving	Adaptation and Flexibility	Evaluation and Justification
Accuracy in Solving Procedures	r_s	0.529 ***	0.474 ***	0.571 ***	0.477 ***
	P	<.001	<.001	<.001	<.001
Efficiency in Problem Solving	r_s	0.525 ***	0.473 ***	0.575 ***	0.563 ***
	p	<.001	<.001	<.001	<.001
Flexibility in Applying Procedures	r_s	0.526 ***	0.504 ***	0.567 ***	0.555 ***
	P	<.001	<.001	<.001	<.001
Appropriateness of Procedures	r_s	0.529 ***	0.473 ***	0.644 ***	0.583 ***
	p	<.001	<.001	<.001	<.001

Notes: Ho: There is no significant relationship between students' procedural fluency and their strategic competence in mathematics learning.

Probability Value Scale: *** p < .001 (Highly Significant); **p<0.01 (Highly Significant); *p<0.05 (Significant); p>0.05 (Not significant)

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue V May 2026

Relationship between Students' Strategic Competence and Conceptual Understanding

Table 5 presents the relationship between students' strategic competence and their conceptual understanding in mathematics, revealing strong and highly significant correlations across all constructs. Spearman's rho values range from 0.488 to 0.656 ($p < .001$), indicating that learners who demonstrate higher levels of problem representation, strategy formulation, adaptation and flexibility, and evaluation and justification also exhibit stronger comprehension, application, connections across topics, and reasoning. The results underscore the importance of strategic competence in fostering conceptual understanding, highlighting that students' ability to plan, monitor, and adjust their problem-solving approaches enhances their grasp of mathematical concepts.

Strategic competence provides a cognitive framework that supports conceptual processing. Students are better equipped to internalize mathematical concepts, identify relevant procedures and ideas, and apply them in a variety of contests when they engage in effective problem representation, carefully formulate strategies, adapt flexibly, and evaluate their solutions. By guiding attention to structure, strategy, and justification, strategic competence lessens cognitive overload, allowing students to focus on reasoning, connecting concepts, and transferring knowledge to new situations.

Problem representation shows strong, highly significant correlations with comprehension of mathematical concepts ($r_s = 0.543$), application of concepts in problem-solving ($r_s = 0.596$), making connections across topics ($r_s = 0.641$), and reasoning and justification ($r_s = 0.597$), all with $p < .001$. The strongest relationship occurs with making connections across topics, demonstrating that students who clearly interpret problem situations are better able to relate mathematical ideas across different contexts. Accurate problem representation helps learners identify relevant information and organize their understanding, thereby supporting conceptual reasoning and the logical explanation of solutions.

Strategy formulation also demonstrates strong correlations with comprehension ($r_s = 0.579$), application ($r_s = 0.576$), making connections across topics ($r_s = 0.655$), and reasoning and justification ($r_s = 0.565$), all with $p < .001$. The strongest correlation with making connections across topics suggests that students who effectively plan and organize their problem-solving approaches are better able to link mathematical concepts. Developing clear strategies enables learners to select appropriate methods and to structure their reasoning, thereby strengthening their conceptual understanding.

Adaptation and flexibility show strong correlations with comprehension ($r_s = 0.498$), application ($r_s = 0.552$), making connections across topics ($r_s = 0.653$), and reasoning and justification ($r_s = 0.550$), all with $p < .001$. The strongest relationship with making connections across topics suggests that students who can adapt their strategies when encountering new problems are better able to transfer and apply mathematical concepts in different situations. This adaptability supports deeper conceptual engagement and flexible thinking in problem-solving.

Evaluation and justification exhibits strong and highly significant correlations with comprehension ($r_s = 0.527$), application ($r_s = 0.488$), making connections across topics ($r_s = 0.656$), and reasoning and justification ($r_s = 0.636$), all with $p < .001$. The strongest associations are with making connections, reasoning, and justification, suggesting that students who critically assess their solutions and explain their reasoning demonstrate stronger conceptual understanding. Reflective evaluation encourages learners to verify results, justify procedures, and connect mathematical ideas more meaningfully.

Recent research supports the strong relationship between strategic competence and conceptual understanding. Students with well-developed problem representation skills can organize and relate mathematical information, facilitating deeper conceptual connections (Torres-Peña et al., 2025). Strategy formulation predicts higher comprehension and application, enabling learners to plan solution pathways and anticipate problem demands (Brod, 2021). Adaptation and flexibility in the use of strategies allow learners to transfer knowledge to new contexts, strengthening integrated understanding across topics (Sharma, 2024). Evaluation and justification enhance reasoning and promote conceptual coherence through reflective problem-solving (Sümen, 2023). Strategic competence bridges procedural execution and higher-order thinking, allowing learners to apply learned procedures meaningfully (Thoe et al., 2022). Collectively, these studies reinforce the highly significant correlations observed in this study and highlight the importance of developing strategic skills to improve conceptual understanding.

The relationship between strategic competence and conceptual understanding can be explained through Information Processing Theory and the Mathematical Proficiency Framework. Efficient encoding, storage, and retrieval of procedures frees cognitive resources for higher-order strategic tasks, such as planning, monitoring, and evaluating solutions (Sridhar & Balamurugan, 2025). Strategic competence, including problem representation, strategy formulation, adaptation, and evaluation, is critical for flexible and reflective problem-solving (Kilpatrick, 2001). Procedural fluency reduces cognitive load, allowing learners to focus on strategy use, which in turn promotes conceptual reasoning, connections, and justification (Kusuma & Retnowati, 2021). Through effective strategies, learners can analyze mathematical situations, make connections among concepts, and justify their reasoning (Sarumaha & Rizkianto, 2022). Empirical studies indicate that students who demonstrate strong strategic competence tend to exhibit deeper conceptual understanding, improved problem-solving ability, and stronger integration of mathematical ideas across contexts (Amalia et al., 2024). These perspectives highlight the important role of strategic competence in supporting and strengthening students' conceptual understanding in mathematics.

Given that making connections, evaluation, and adaptation exhibit the strongest correlations with strategic competence, educators should implement activities that encourage students to represent problems clearly, formulate effective strategies, adjust approaches for novel tasks, and justify their solutions. Strategies such as reflective problem-solving, strategy journals, guided peer discussions, and real-world problem modeling can foster adaptive thinking, strategic planning, and critical evaluation. By strengthening strategic competence, students not only enhance their procedural application but also develop deeper conceptual understanding, supporting holistic mathematical proficiency across diverse problem contexts.

Table 5 Significant Relationship between Students' Strategic Competence and their Conceptual Understanding

Variables		Problem Representation	Strategy Formulation	Adaptation and Flexibility	Evaluation and Justification
Comprehension of Mathematical Concepts	r_s	0.543 ***	0.579 ***	0.498 ***	0.527 ***
	p	<.001	<.001	<.001	<.001
Application of Concepts in Problem Solving	r_s	0.596 ***	0.576 ***	0.552 ***	0.488 ***
	p	<.001	<.001	<.001	<.001
Making Connections Across Topics	r_s	0.641 ***	0.655 ***	0.653 ***	0.656 ***
	p	<.001	<.001	<.001	<.001
Reasoning and Justification	r_s	0.597 ***	0.565 ***	0.550 ***	0.636 ***
	p	<.001	<.001	<.001	<.001

Notes: Ho: There is no significant relationship between students' strategic competence and conceptual understanding in mathematics learning.

Probability Value Scale: *** $p < .001$ (Highly Significant); **$p<0.01$ (Highly Significant); *$p<0.05$ (Significant); $p>0.05$ (Not significant)

Significant Relationship between Students' Procedural Fluency and Conceptual Understanding

Table 6 presents the relationship between students' procedural fluency and conceptual understanding in mathematics, showing strong and highly significant correlations across all constructs. Spearman's rho values range from 0.507 to 0.709 ($p < .001$), indicating that students who demonstrate higher accuracy, efficiency, flexibility, and appropriateness in their procedural execution also exhibit higher levels of comprehension, application, connections across topics, and reasoning. These results highlight that procedural fluency provides a foundational platform for cognitive engagement, supporting conceptual mastery and demonstrating the interdependent nature of procedural skills and conceptual understanding in mathematics learning.

Procedural fluency in this study encompasses accuracy in following procedures, efficiency in problem-solving, flexibility in applying procedures, and appropriateness of methods. Conceptual understanding includes comprehension of mathematical concepts, application in problem-solving, making connections across topics, and reasoning and justification. The analysis shows that students' ability to perform operations correctly and adaptively underpins their capacity to internalize mathematical ideas and transfer knowledge to new problem contexts. This relationship aligns with cognitive theories suggesting that efficient processing of procedural steps enhances memory encoding, problem representation, and knowledge transfer, thereby facilitating deeper conceptual understanding.

Accuracy in solving procedures shows strong and highly significant correlations with all dimensions of conceptual understanding, including comprehension of mathematical concepts ($r_s = 0.594$), application of concepts in problem-solving ($r_s = 0.709$), making connections across topics ($r_s = 0.633$), and reasoning and justification ($r_s = 0.593$), all with $p < .001$. The strongest relationship occurs with the application of concepts, indicating that students who execute procedures accurately are more capable of applying mathematical ideas effectively when solving problems. Accurate procedural performance helps ensure that solution steps are correct, allowing learners to focus on understanding relationships among concepts and explaining their reasoning.

Efficiency in problem solving also demonstrates strong and highly significant correlations with comprehension ($r_s = 0.546$), application ($r_s = 0.572$), making connections across topics ($r_s = 0.565$), and reasoning and justification ($r_s = 0.557$), all with $p < .001$. These findings suggest that students who solve problems efficiently, balancing speed and accuracy, can process mathematical tasks more effectively and devote greater attention to interpreting results and connecting ideas. Efficient procedural performance, therefore, supports both conceptual comprehension and logical reasoning in mathematics.

Flexibility in applying procedures shows moderate-to-strong correlations with comprehension ($r_s = 0.507$) and application ($r_s = 0.521$), and stronger relationships with making connections across topics ($r_s = 0.640$) and reasoning and justification ($r_s = 0.682$), all with $p < .001$. The highest correlation with reasoning and justification indicates that students who can adapt procedures to different situations are better able to explain their thinking and justify their solutions. Procedural flexibility encourages deeper conceptual engagement, as learners must understand how and why different procedures work across varying contexts.

The appropriateness of procedures demonstrates strong, highly significant correlations with comprehension ($r_s = 0.539$), application ($r_s = 0.621$), making connections across topics ($r_s = 0.652$), and reasoning and justification ($r_s = 0.663$), all with $p < .001$. The strongest associations occur with making connections and reasoning, suggesting that selecting the most suitable procedure for a problem strengthens students' ability to relate concepts and justify their solutions. Choosing appropriate methods reflects deeper mathematical understanding because learners must evaluate problem conditions and determine which strategies best support accurate and meaningful solutions.

Recent studies support these findings. Procedural fluency facilitates higher-order conceptual understanding, particularly when students adapt strategies to diverse problems (Gradini et al., 2025). Accurate and efficient procedural execution predicts stronger problem-solving application and reasoning skills (Zhang et al., 2022). Flexibility in procedural strategies enhances learners' capacity to connect concepts, while the appropriate selection of procedures supports comprehension and justification (Rukshana, 2025). Students with well-developed procedural skills internalize concepts more effectively (Rif'at, 2024), and integrating procedural and conceptual practice fosters holistic mathematical competence (Siregar, 2025).

Information Processing Theory (Atkinson & Shiffrin, 1968) posits that cognitive processes involve encoding, storing, and retrieving information, and procedural fluency enhances memory encoding, retrieval efficiency, and cognitive automation. Students who execute mathematical procedures accurately and efficiently can focus cognitive resources on understanding underlying principles. Constructivist Learning Theory (Piaget, 1972) suggests that learners actively construct knowledge by integrating new information with prior understanding. Conceptual understanding develops as students internalize mathematical concepts, make connections, and justify solutions (Biber, 2023). Together, these theories explain how procedural fluency (cognitive scaffolding) supports conceptual understanding (active knowledge construction) (Zeng, 2025). Recent studies further emphasize that procedural mastery enhances conceptual reasoning and promotes active engagement with problem-solving (Naz, 2025).

Procedural fluency is critical for developing students' conceptual understanding, particularly in higher-order constructs like flexibility, appropriateness, reasoning, and making connections. Instructional strategies should focus on cultivating adaptive problem-solving and contextually appropriate procedures, alongside reflective activities that require justification and strategic application. By strengthening these procedural dimensions, educators can bridge the gap between performing mathematical operations and internalizing concepts, fostering holistic mathematical competence, and preparing students for complex problem-solving scenarios and the underlying mathematical concepts.

Table 6 Significant Relationship between Students' Procedural Fluency and Conceptual Understanding

Variables		Comprehension of Mathematical Concepts	Application of Concepts in Problem-Solving	Making Connections Across Topics	Reasoning and Justification
Accuracy in Solving Procedures	r_s	0.594 ***	0.709 ***	0.633 ***	0.593 ***
	p	<.001	<.001	<.001	<.001
Efficiency in Problem Solving	r_s	0.546 ***	0.572 ***	0.565 ***	0.557 ***
	p	<.001	<.001	<.001	<.001
Flexibility in Applying Procedures	r_s	0.507 ***	0.521 ***	0.640 ***	0.682 ***
	p	<.001	<.001	<.001	<.001
Appropriateness of Procedures	r_s	0.539 ***	0.621 ***	0.652 ***	0.663 ***
	p	<.001	<.001	<.001	<.001

Notes: Ho: There is no significant relationship between students' procedural fluency and conceptual understanding in mathematics learning.

Probability Value Scale: *** p < .001 (Highly Significant); **p<0.01 (Highly Significant); *p<0.05 (Significant); p>0.05 (Not significant)

Predictor of Students' Conceptual Understanding in Mathematics

A multiple linear regression analysis was conducted to determine whether selected dimensions of students' procedural fluency and strategic competence significantly predict students' conceptual understanding in mathematics. In this model, efficiency in problem-solving, flexibility in applying procedures, and the appropriateness of procedures represented the procedural fluency variables. In contrast, problem representation, strategy formulation, and evaluation and justification represented components of strategic competence. Students' conceptual understanding served as the dependent variable. The analysis aimed to examine how these cognitive dimensions collectively influence students' ability to comprehend mathematical concepts, establish relationships among ideas, and apply knowledge meaningfully.

The results indicate that the regression model demonstrates a highly significant relationship between the predictors and students' conceptual understanding. The adjusted R^2 value of 80.59% shows that approximately 80.59% of the variation in students' conceptual understanding can be explained by the combined influence of procedural fluency and strategic competence variables. This proportion is considered very high, suggesting that the model has strong explanatory power. Furthermore, the model is statistically significant ($F = 94.43$, $p < .001$), confirming that the predictors, when taken together, provide a meaningful and reliable explanation of students' conceptual understanding and that the results are unlikely due to chance.

Examining the individual predictors, all variables were found to have a statistically significant, positive influence on students' conceptual understanding. Among the predictors, appropriateness of procedures emerged as the strongest predictor ($\beta = 0.23$, $t = 5.27$, $p < .001$). This suggests that students who can select and apply appropriate mathematical procedures are more likely to develop a deeper understanding of the underlying concepts. When learners choose suitable procedures, it reflects not only procedural knowledge but also an awareness of why certain methods are effective, which strengthens conceptual clarity.

Problem representation was also identified as a strong predictor ($\beta = 0.20$, $t = 5.05$, $p < .001$). This indicates that students who can accurately interpret and represent mathematical problems tend to have higher conceptual understanding. The ability to translate problems into mathematical forms, such as diagrams, symbols, or equations, enables learners to organize information and recognize relationships among concepts, thereby enhancing comprehension. Similarly, flexibility in applying procedures showed a significant positive effect ($\beta = 0.16$, $t = 3.06$, $p = 0.003$), suggesting that students who can adapt their procedures across different contexts are better able to understand mathematical ideas at a deeper level.

Efficiency in problem solving ($\beta = 0.12$, $t = 3.95$, $p < .001$) and strategy formulation ($\beta = 0.12$, $t = 2.58$, $p = 0.011$) were also significant predictors of conceptual understanding. These findings imply that students who can solve problems efficiently and plan appropriate strategies tend to develop clearer and more structured conceptual knowledge. Efficient problem solvers focus on essential aspects of a task, while effective strategy formulation supports logical thinking and organized reasoning. Evaluation and justification likewise contributed significantly ($\beta = 0.09$, $t = 2.18$, $p = 0.030$), indicating that students who reflect on and justify their solutions are more likely to solidify their conceptual understanding, as this process requires them to explain and validate the relationships among mathematical ideas.

Recent studies in mathematics education strongly support these findings. Research shows that conceptual understanding is enhanced when students engage in meaningful problem-solving processes that require them to select appropriate procedures and justify their reasoning (Fielding & Makar, 2022). A recent study found that students' conceptual understanding improves when they actively engage in mathematical tasks that involve explanation, representation, and reasoning across multiple topics (Prediger, 2022). Similarly, a systematic review emphasized that conceptual learning is closely linked to students' ability to analyze, interpret, and apply mathematical ideas flexibly, rather than relying solely on routine procedures (Tañola & Lomibao, 2024).

Moreover, related studies highlight the importance of problem representation and strategic processes in developing conceptual understanding (Benedek et al., 2023). According to recent trends in mathematics education, learners' ability to represent problems using symbolic, graphical, or contextual forms significantly shapes their reasoning and understanding of mathematical concepts (Mutodi & Mosimege, 2021). This supports the present finding that problem representation is one of the strongest predictors of conceptual understanding. In addition, studies on problem-solving models indicate that conceptual understanding develops when students engage in structured problem-solving approaches that integrate reasoning, flexibility, and justification (Elhilal, 2025).

Furthermore, recent empirical research underscores that procedural fluency and conceptual understanding are interconnected rather than separate constructs (Keazer & Phaiah, 2023). Learners who demonstrate efficiency and flexibility in solving problems tend to develop deeper conceptual knowledge because they can focus on relationships among concepts rather than merely executing steps (Arsyad & Syakhrani, 2024). This explains why efficiency, flexibility, and appropriateness of procedures significantly contributed to conceptual understanding in the present study. Additionally, studies have shown that metacognitive processes, such as evaluating and justifying solutions, play a critical role in strengthening conceptual learning by encouraging students to reflect on their reasoning and validate their understanding (Rivas et al., 2022).

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue V May 2026

Overall, the results suggest that students' conceptual understanding is significantly shaped by both procedural fluency and strategic competence. When students can efficiently solve problems, apply procedures flexibly and appropriately, represent problems clearly, formulate strategies, and justify their solutions, they are more likely to develop a deep and connected understanding of mathematical concepts. These findings highlight that conceptual understanding is not developed in isolation but rather results from the integration of multiple cognitive skills.

The findings have important implications for mathematics instruction. Teachers should design learning experiences that integrate procedural fluency with strategic thinking skills. Instructional approaches such as problem-based learning, use of multiple representations, guided questioning, and reflective activities can help students strengthen both their procedural and conceptual knowledge. By fostering efficiency, flexibility, and reasoning in problem solving, educators can support the development of deeper conceptual understanding, ultimately improving students' overall mathematical proficiency.

Table 7

Predictors	Coef (β)	SE Coef	t- value	p-value
(Constant)	0.16	0.14	1.124	0.263
Efficiency in Problem Solving	0.12	0.03	3.95	<.001
Flexibility in Applying Procedures	0.16	0.52	3.06	0.003
Appropriateness of Procedures	0.23	0.04	5.27	<.001
Problem Representation	0.20	0.04	5.05	<.001
Strategy Formulation	0.12	0.05	2.58	0.011
Evaluation and Justification	0.09	0.04	2.18	0.030
Adjusted r^2 80.59%				
F value 94.43				
p-value < .001				
Students' Conceptual Understanding = 0.16+ 0.12* Efficiency in Problem Solving+0.16* Flexibility in Applying Procedures+0.23*Appropriateness of Procedures+0.20*Problem Representation+0.12*Strategy Formulation+0.09*Strategy Formulation				

Mediation Analysis on the Effect of Students' Strategic Competence in the Relationship between their Procedural Fluency and Conceptual Understanding in Mathematics

A mediation analysis was conducted to determine whether students' strategic competence mediates the relationship between procedural fluency and conceptual understanding in mathematics learning (Table 8). The hypothesis tested in this analysis was that strategic competence significantly mediates the relationship between procedural fluency and conceptual understanding among students. The analysis examined whether procedural fluency influences conceptual understanding directly or indirectly through the development of students' strategic competence. The results revealed that the total effect of procedural fluency on conceptual understanding was statistically significant, [B = 0.900, SE = 0.0419, 95% CI, 0.818, 0.983, Z = 21.48, p < .001]. This indicates that higher levels of procedural fluency are strongly associated with greater conceptual understanding among students in mathematics.

The direct effect of procedural fluency on conceptual understanding remained statistically significant even after controlling for the mediating role of strategic competence, [B = 0.536, SE = 0.0551, 95% CI, 0.428,

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue V May 2026

0.644, Z = 9.71, p < .001]. This finding suggests that procedural fluency independently contributes to students' conceptual understanding. Students who can perform mathematical procedures accurately and efficiently are better able to understand underlying principles and apply them in problem-solving situations.

At the same time, the indirect effect through strategic competence was also statistically significant, [B = 0.365, SE = 0.0480, 95% CI, 0.271, 0.459, Z = 7.60, p < .001]. This indicates that strategic competence significantly mediates the relationship between procedural fluency and conceptual understanding. The mediation results further show that 40.5% of the total effect occurs indirectly through strategic competence, while 59.5% remains direct, indicating a partial mediation effect. This suggests that procedural fluency enhances conceptual understanding not only through direct mastery of mathematical procedures but also by strengthening students' ability to develop and apply effective problem-solving strategies.

The path estimates further support this mediation relationship. Procedural fluency significantly predicted strategic competence, [B = 0.758, SE = 0.0470, 95% CI, 0.666, 0.850, Z = 16.14, p < .001], indicating that students with stronger procedural skills are more capable of formulating and adapting strategies when solving mathematical problems. In turn, strategic competence significantly predicted conceptual understanding, B = 0.481, SE = 0.0559, 95% CI, 0.372, 0.591, Z = 8.61, p < .001], suggesting that students who can represent problems clearly, plan solution strategies, and evaluate their reasoning tend to demonstrate deeper understanding of mathematical concepts.

These findings are consistent with recent studies that emphasize the interconnectedness of procedural fluency, strategic competence, and conceptual understanding in mathematics learning. Procedural fluency provides a foundational skill set that allows students to perform mathematical operations accurately and efficiently, enabling them to focus on higher-level reasoning processes (Go et al., 2023). Strategic competence, on the other hand, enables students to formulate strategies, represent mathematical situations, and evaluate solutions, which strengthens their ability to understand and apply mathematical concepts across different contexts (Ukobizaba et al., 2021). Similarly, research indicates that students who engage in strategic problem-solving demonstrate stronger conceptual understanding by actively connecting procedures to underlying mathematical ideas (Vale et al., 2023).

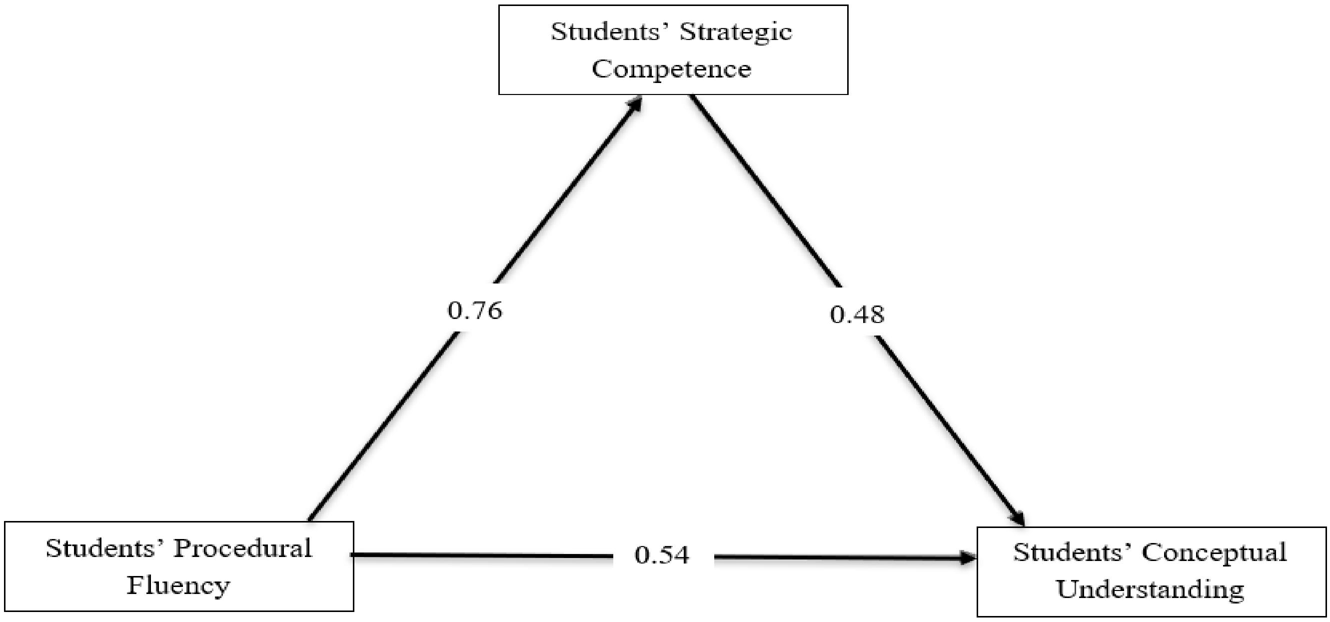
Overall, the mediation analysis highlights the important role of strategic competence as a pathway through which procedural fluency contributes to conceptual understanding. While procedural skills directly support conceptual learning, their impact is stronger when students can use them strategically in problem-solving situations. These findings suggest that mathematics instruction should emphasize not only procedural accuracy but also strategic thinking. Learning activities that encourage students to represent problems, formulate strategies, adapt approaches, and justify their reasoning may help strengthen strategic competence and ultimately enhance students' conceptual understanding of mathematics.

Table 8 Mediation analysis on the effect of students' strategic competence in the relationship between procedural fluency and conceptual understanding in mathematics learning.

Effect			**Label**	**Estimate**	**SE**	**95% Confidence Interval**		**Z**	**p**	**% Mediation**
						Lower	**Upper**			
Indirect			a x b	0.365	0.0480	0.271	0.459	7.60	< .001	40.5
Direct			c	0.536	0.0551	0.428	0.644	9.71	< .001	59.5
Total			c+a x b	0.900	0.0419	0.818	0.983	21.48	< .001	100.0
Path Estimates										

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue V May 2026

Students' Procedural Fluency	→	Procedural Fluency	a	0.758	0.0470	0.666	00.850	16.14	< .001	
Students' Conceptual Understanding	→	Conceptual Understanding	b	0.481	0.0559	0.372	0.591	8.61	< .001	
Students' Strategic Competence	→	Strategic Competence	c	0.536	0.0551	0.428	0.644	9.71	< .001	



The diagram illustrates the mediation model showing the effect of students' strategic competence on the relationship between procedural fluency and conceptual understanding in mathematics learning. The path coefficients indicate the strength of the relationships among the three variables, providing a clearer explanation of how procedural skills may influence deeper mathematical understanding. These findings imply that while procedural fluency alone contributes to conceptual learning, its effect is stronger when students can strategically apply their procedural knowledge to solve mathematical problems. Therefore, the diagram highlights the important role of strategic competence as a significant pathway in strengthening students' mathematical understanding.

CONCLUSIONS AND RECOMMENDATIONS

Based on the findings of the study, it can be concluded that students generally possess high levels of procedural fluency, conceptual understanding, and strategic competence in mathematics. The results indicate that students are capable of accurately performing mathematical procedures, understanding and applying mathematical concepts, and effectively solving problems through appropriate strategies. Furthermore, the study revealed that procedural fluency is closely associated with deeper conceptual understanding, while strong procedural skills also contribute to higher levels of strategic competence. The findings further suggest that connecting mathematical ideas with accurate procedural performance enhances students' ability to formulate and evaluate effective problem-solving strategies. Overall, the study highlights that procedural fluency, conceptual understanding, and strategic competence are interconnected and collectively contribute to students' holistic mathematical proficiency. Effective mathematics instruction, therefore, plays a vital role in promoting the balanced development of these essential mathematical skills.

In light of the findings and conclusions of the study, it is recommended that mathematics teachers continue to strengthen students' procedural fluency through varied practice activities that promote accuracy and efficiency in mathematical procedures. Teachers are also encouraged to incorporate problem-solving activities that develop students' strategic competence by allowing them to represent problems, formulate strategies, adapt solution methods, and evaluate their answers. Furthermore, instructional strategies that emphasize conceptual explanations, meaningful applications, and a deeper understanding of mathematical principles should be consistently implemented to enhance conceptual understanding. Learning tasks should likewise be designed to integrate procedural skills with strategic problem-solving approaches, enabling students to apply their knowledge effectively. In addition, teachers may encourage students to explain and justify their reasoning in order to further strengthen both conceptual understanding and strategic competence. Finally, classroom instruction should continuously integrate procedural practice with conceptual discussions to support the simultaneous development of procedural fluency and conceptual understanding among learners.

REFERENCES

1. Amalia, L., Makmuri, M., & El Hakim, L. (2024). Learning design: To improve mathematical problem-solving skills using a contextual approach. JIIP-Jurnal Ilmiah Ilmu Pendidikan, 7(3), 2353-2366. https://www.jiip.stkipyapisdompu.ac.id/jiip/index.php/JIIP/article/view/3455
2. Andal, S. G. B., & Andrade, R. R. (2022). Exploring students' procedural fluency and written adaptive reasoning skills in solving open-ended problems. International Journal of Science, Technology, Engineering and Mathematics, 2(1), 1-25. https://www.neliti.com/publications/356453/exploring-students-procedural-fluency-and-written-adaptive-reasoning-skills-in-s
3. Annurwanda, P., & Friantini, R. N. (2022). Mathematical induction proofing: Procedural fluency reviewed from the creative thinking level of mathematics students. Jurnal Riset Pendidikan Matematika, 9(1), 22-35. https://journal.lppmunindra.ac.id/index.php/Formatif/article/view/9907
4. Applebaum, M. (2024). Enhancing critical thinking in pre-service mathematics teachers: Bridging procedural fluency and conceptual understanding. Математика плюс, 32(3), 58-66. https://www.ceeol.com/search/article-detail?id=1276653
5. Arsyad, M., & Syakhrani, A. W. (2024). The efficiency of using visual learning media in improving the understanding of science concepts in elementary school students. Indonesian Journal of Education (INJOE), 4(1), 775-787. https://felifa.net/index.php/INJOE/article/view/234
6. Atkinson, R. C., & Shiffrin, R. M. (1968). Human memory: A proposed system and its control processes. In K. W. Spence & J. T. Spence (Eds.), The psychology of learning and motivation (Vol. 2, pp. 89–195). Academic Press. https://tinyurl.com/39e5dbv5
7. Benedek, M., Beaty, R. E., Schacter, D. L., & Kenett, Y. N. (2023). The role of memory in creative ideation. Nature Reviews Psychology, 2(4), 246-257. http://nature.com/articles/s44159-023-00158-z
8. Biber, M. (2023). Constructing Mental Models: Mathematical Communication and Discourses in Conceptual Understanding. Necmettin Erbakan Üniversitesi Ereğli Eğitim Fakültesi Dergisi, 5(Özel Sayı), 271-305. https://dergipark.org.tr/en/pub/neueefd/article/1341680?issue_id=80589
9. Brancaccio, A., De Chiusole, D., & Stefanutti, L. (2023). Algorithms for the adaptive assessment of procedural knowledge and skills. Behavior Research Methods, 55(7), 3929-3951. https://link.springer.com/article/10.3758/s13428-022-01998-y
10. Brod, G. (2021). Predicting as a learning strategy. Psychonomic Bulletin & Review, 28(6), 1839-1847. https://link.springer.com/article/10.3758/s13423-021-01904-1
11. Bürgler, S., Hoyle, R. H., & Hennecke, M. (2021). Flexibility in using self-regulatory strategies to manage self-control conflicts: The role of metacognitive knowledge, strategy repertoire, and feedback monitoring. European Journal of Personality, 35(6), 861-880. https://journals.sagepub.com/doi/abs/10.1177/0890207021992907
12. Cabuquin, J., & Abocejo, F. (2024). Conceptual and procedural understanding in the division of algebraic fractions. Recoletos Multidisciplinary Research Journal, 12(1), 225-240. https://tinyurl.com/mwh5en57
13. Copur-Gencturk, Y. (2021). Teachers' conceptual understanding of fraction operations: results from a national sample of elementary school teachers. Educational Studies in Mathematics, 107(3), 525-545. https://link.springer.com/article/10.1007/s10649-021-10033-4

14. Corrêa, P. D., & Haslam, D. (2021). Mathematical Proficiency as the Basis for Assessment: A Literature Review and Its Potentialities. Mathematics Teaching Research Journal, 12(4), 3-20. https://tinyurl.com/5epn7n5h
15. Creswell, J. W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research (4th ed., p. 337). Pearson Education, Inc. https://www.scribd.com/document/713796239/Creswell-2012-educational-research
16. Dingman, S., Teuscher, D., Olson, T. A., & Kasmer, L. A. (2021). Conceptualizing curricular reasoning: A framework for examining mathematics teachers' curricular decisions. Investigations in Mathematics Learning, 13(4), 267-286. https://www.tandfonline.com/doi/abs/10.1080/19477503.2021.1981742
17. Diputra, K. S., Agustika, G. N. S., Utami, I. A. M. I., Julianto, P., & Arifuddin, A. (2025). Investigating mathematical proficiency of elementary school students: A foundation for effective learning models. Edelweiss Applied Science and Technology, 9(5), 314-323. Investigating mathematical proficiency of elementary school students: A foundation for effective learning models
18. Dood, A. J., & Watts, F. M. (2022). Students' strategies, struggles, and successes with mechanism problem solving in organic chemistry: a scoping review of the research literature. Journal of Chemical Education, 100(1), 53-68. https://pubs.acs.org/doi/abs/10.1021/acs.jchemed.2c00572
19. Elhilal, A. (2025). Digital conceptual mapping for enhancing mathematical concept formation and creative mathematical problem-solving through cognitive flexibility skills: a mixed methods study. Cogent Education, 12(1), 2494945.https://www.tandfonline.com/doi/full/10.1080/2331186X.2025.2494945
20. Elvi, M. (2025). MATHEMATICAL PROFICIENCY: AN ANALYSIS OF PROSPECTIVE TEACHERS'ABILITY TO SOLVE MATHEMATICAL PROBLEMS. Jurnal Pendidikan Sang Surya, 11(1), 774-782. https://jurnal.umbulukumba.ac.id/index.php/jpss/article/view/449
21. Fielding, Jill, and Katie Makar. "Challenging conceptual understanding in a complex system: Supporting young students to address extended mathematical inquiry problems." Instructional Science 50.1 (2022): 35-61. https://link.springer.com/article/10.1007/s11251-021-09564-3
22. Go, M. C., Cris, M., & Go, J. (2023). Enhancing mathematical proficiency assessment: Insights from mathematics teachers. Science International, 35(6), 773-780. https://www.researchgate.net/profile/Mary-Cris-Go/publication/375828757_ENHANCING_MATHEMATICAL_PROFICIENCY_ASSESSMENT_INSIGHTS_FROM_MATHEMATICS_TEACHERS/links/655e74163fa26f66f420954c/ENHANCING-MATHEMATICAL-PROFICIENCY-ASSESSMENT-INSIGHTS-FROM-MATHEMATICS-TEACHERS.pdf
23. Gradini, E., Noviani, J., & Ulya, K. (2025). Fostering higher-order thinking skills in mathematics education: strategies, challenges, and classroom practices. Prisma Sains: Jurnal Pengkajian Ilmu Dan Pembelajaran Matematika Dan IPA IKIP Mataram, 13(2), 135-163. https://ojspanel.undikma.ac.id/index.php/prismasains/article/view/15099
24. Haque, M. N. (2024). The role of multiple representations and attitudes in enhancing statistical and mathematical learning. Smart Internet of Things, 1(4), 298-312. https://www.siot.reapress.com/journal/article/view/52
25. Hariri, D. D., & Kania, N. (2025). Mapping Critical Thinking Skills through Newman's Error Analysis in Secondary Students' Problem-Solving. Jurnal Pendidikan MIPA, 26(3), 1464-1478. https://jpmipa.fkip.unila.ac.id/index.php/jpmipa/article/view/782
26. Hickendorff, M. (2022). Flexibility and adaptivity in arithmetic strategy use: What children know and what they show. Journal of Numerical Cognition, 8(3), 367-381. https://jnc.psychopen.eu/index.php/jnc/article/view/7277
27. Hong, W., Star, J. R., Liu, R. D., Jiang, R., & Fu, X. (2023). A systematic review of mathematical flexibility: Concepts, measurements, and related research. Educational Psychology Review, 35(4), 104. https://link.springer.com/article/10.1007/s10648-023-09825-2 https://tinyurl.com/y84hjhpx
28. Hurrell, D. (2021). Conceptual knowledge or procedural knowledge or conceptual knowledge and procedural knowledge: Why the conjunction is important to teachers. Australian Journal of Teacher Education (Online), 46(2), 57-71. https://search.informit.org/doi/abs/10.3316/informit.757709794375494

29. Karatas, S. (2022). The Impacts of Supporting Productive Struggle Teaching Practice on Students' Conceptual Understanding, Procedural Fluency, and Strategic Competence: The Case of Quadratic Functions (Doctoral dissertation, The University of Texas Rio Grande Valley). https://link.springer.com/article/10.1007/s10857-023-09590-7
30. Keazer, L., & Phaiah, J. (2023). Analyzing prospective elementary teachers' evidence of conceptual understanding and procedural fluency. Investigations in Mathematics Learning, 15(2), 135-148. https://www.tandfonline.com/doi/abs/10.1080/19477503.2022.2139112
31. Khaerunnisa, E., Aliyudin, A., Ruhiat, Y., & Santosa, C. A. H. F. (2025). PRODUCTIVE DISPOSITION DEVELOPMENT MODEL BASED ON CONCEPT UNDERSTANDING WITH MEDIATORS OF PROCEDURAL FLUENCY, STRATEGIC COMPETENCE, ADAPTIVE REASONING. AKSIOMA: Jurnal Program Studi Pendidikan Matematika, 14(1). https://ojs.fkip.ummetro.ac.id/index.php/matematika/article/view/11442
32. Kholid, M. N., Imawati, A., Swastika, A., Maharani, S., & Pradana, L. N. (2021, February). How are Students' Conceptual Understanding for Solving Mathematical Problem?. In Journal of Physics: Conference Series (Vol. 1776, No. 1, p. 012018). IOP Publishing. https://tinyurl.com/39vx5zet
33. Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. National Academy Press. https://tinyurl.com/8794628p
34. Kusuma, I. A., & Retnowati, E. (2021, March). Designs of faded-example to increase problem solving skills and procedural fluency in algebraic division. In Journal of Physics: Conference Series (Vol. 1806, No. 1, p. 012109). IOP Publishing.
35. Landreth, S. J., & Young, C. (2021). Developing fluency and comprehension with the secondary fluency routine. The Journal of Educational Research, 114(3), 252-262. https://www.tandfonline.com/doi/abs/10.1080/00220671.2021.1910475
36. Landreth, S. J., & Young, C. (2021). Developing fluency and comprehension with the secondary fluency routine. The Journal of Educational Research, 114(3), 252-262. https://www.tandfonline.com/doi/abs/10.1080/00220671.2021.1910475
37. Manandhar, N. K., Pant, B. P., & Dawadi, S. D. (2022). Conceptual and procedural knowledge of students of Nepal in algebra: A mixed method study. Contemporary Mathematics and Science Education, 3(1), 1-10. https://tinyurl.com/4y28bby3
38. Mohamed Elsayed, S. A. (2022). The effectiveness of learning mathematics according to the STEM approach in developing the mathematical proficiency of second graders of the intermediate school. Education Research International, 2022(1), 5206476. https://tinyurl.com/ymprwmk2
39. Mutodi, P., & Mosimege, M. (2021). Learning mathematical symbolization: Conceptual challenges and instructional strategies in secondary schools. Bolema: Boletim de Educação Matemática, 35, 1180-1199. https://www.scielo.br/j/bolema/a/nHVdpBMTykbcxB4MM7rPD8c/?format=html&lang=en
40. Naz, B., & Qayyum, A. (2025). Effectiveness of Problem-Solving as a Teaching Strategy in Enhancing Students' Conceptual Understanding and Critical Thinking in Mathematics Education. Research Journal for Social Affairs, 3(6), 693-702. https://rjsaonline.com/journals/index.php/rjsa/article/view/500
41. Ncube, M., & Luneta, K. (2025). Concept-based instruction: Improving learner performance in mathematics through conceptual understanding. Pythagoras-Journal of the Association for Mathematics Education of South Africa, 46(1), 815. https://journals.co.za/doi/abs/10.4102/pythagoras.v46i1.815
42. Nurtamam, M. E., & Jannah, A. N. (2025). A systematic qualitative review of teachers' strategies in enhancing mathematical reasoning in elementary schools. Jurnal Obsesi: Jurnal Pendidikan Anak Usia Dini, 9(2), 553-562. https://tinyurl.com/mura6sjx
43. Öz, T., & Işık, A. (2024). Exploring mathematical reasoning skills of middle school students. Thinking Skills and Creativity, 53, 101612. https://www.sciencedirect.com/science/article/abs/pii/S1871187124001500
44. Piaget, J. (1972). The principles of genetic epistemology. Basic Books. https://tinyurl.com/ydr5tnxw
45. Pratt, D. J. (2023). The impact of automaticity training in multiplication facts on problem solving ability (Doctoral dissertation, University of Otago). https://ourarchive.otago.ac.nz/esploro/outputs/doctoral/The-impact-of-automaticity-training-in/9926478241801891

46. Prediger, S. (2022, February). Enhancing language for developing conceptual understanding: A research journey connecting different research approaches. In Twelfth Congress of the European Society for Research in Mathematics Education (CERME12). https://hal.science/hal-03756062/
47. Rif'at, M., Sudiansyah, S., & Imama, K. (2024). Role of visual abilities in mathematics learning: An analysis of conceptual representation. Al-Jabar: Jurnal Pendidikan Matematika, 15(1), 87-97. https://ejournal.radenintan.ac.id/index.php/al-jabar/article/view/22406
48. Rivas, S. F., Saiz, C., & Ossa, C. (2022). Metacognitive strategies and development of critical thinking in higher education. Frontiers in psychology, 13, 913219.https://www.frontiersin.org/journals/psychology/articles/10.3389/fpsyg.2022.913219/full?utm_source=chatgpt.com
49. Rukshana, M. M. M., Gunavathi, M. S., & Prithiyangadevi, M. S. (2025). FROM PROCEDURES TO CONCEPTUAL INSIGHT: A REVIEW OF TEACHING STRATEGIES IN MATHEMATICS. Contemporary Techniques in Math Education, 3. https://www.researchgate.net/profile/A-Dinesh-Kumar-2/publication/391369291_Contemporary_Techniques_in_Math_Education/links/68144683bfbe974b23c1b70d/Contemporary-Techniques-in-Math-Education.pdf#page=147
50. Santosa, Y. T., Kholid, M. N., Ishartono, N., Fitriyya, M., & Junaedi, I. (2025). Toward a Theoretical Model of Deep Mathematical Thinking: Integrating Deep Learning and Mathematical Reasoning Frameworks. Journal of Deep Learning, 109-126. https://journals2.ums.ac.id/jdl/article/view/11142
51. Sarumaha, Y. A., & Rizkianto, I. (2022). Promoting mathematical justification through realistic mathematics education classroom. Jurnal Pendidikan Matematika (JUPITEK), 5(2), 83-94. https://tinyurl.com/3yejs9c8
52. Schulz, A. (2024). Assessing student teachers' procedural fluency and strategic competence in operating and mathematizing with natural and rational numbers. Journal of Mathematics Teacher Education, 27(6), 981–1008. https://tinyurl.com/7tccm79p
53. Sharma, S. (2024). Enhancing inclusive learning environments: Strategies for curriculum adaptation and modification. Future of special education in India, 109, 121. https://www.researchgate.net/profile/Geetanjali-Sharma-18/publication/380266832_978-93-5879-742-8_ebook_1/links/66332a4d06ea3d0b741f9b4c/978-93-5879-742-8-ebook-1.pdf#page=122
54. Sidhu, A., Bhalla, P., & Zafar, S. (2021). Mediating effect and review of its statistical measures. Empir Econ Lett, 20(4), 29-40. Mediating-Effect-and-Review-of-its-Statistical-Measures.pdf
55. Singh, C., Maries, A., Heller, K., & Heller, P. (2023). Instructional strategies that foster effective problem-solving. arXiv preprint arXiv:2304.05585. https://arxiv.org/abs/2304.05585
56. Siregar, T. (2025). Integrating GeoGebra in mathematics education: Enhancing pedagogical practices among teachers and lecturers. Yerizon, Integrating GeoGebra in Mathematics Education: Enhancing Pedagogical Practices among Teachers and Lecturers (October 16, 2025). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5613052
57. Siregar, T. (2025). The effectiveness of the discovery learning model in enhancing students' mathematical problem-solving skills. https://www.preprints.org/manuscript/202510.1549
58. Skylark, W. J. (2021). More is easier? Testing the role of fluency in the more-credible effect. Judgment and Decision Making, 16(3), 638-686. https://www.cambridge.org/core/journals/judgment-and-decision-making/article/more-is-easier-testing-the-role-of-fluency-in-the-morecredible-effect/0C28BC1EA5CE6F6DC3C8C3C4FD09BE99
59. Sridhar, M. K., & Balamurugan, M. S. (2025). COGNITIVE GROWTH AND MATHEMATICAL COMPREHENSION: A REVIEW OF CONTEMPORARY EDUCATIONAL THEORIES. Contemporary Techniques in Math Education. https://www.researchgate.net/profile/A-Dinesh-Kumar-2/publication/391369291_Contemporary_Techniques_in_Math_Education/links/68144683bfbe974b23c1b70d/Contemporary-Techniques-in-Math-Education.pdf#page=49
60. Star, J. R., Tuomela, D., Joglar-Prieto, N., Hästö, P., Palkki, R., Abánades, M. Á., ... & Liu, R. D. (2022). Exploring students' procedural flexibility in three countries. International Journal of STEM Education, 9(1), 4. Öz, T., & Çiftci, Z. (2024). Mathematical Reasoning Activity: Compare, Generalize and Justify. Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 18(2), 291-323. https://dergipark.org.tr/en/pub/balikesirnef/article/1506921

61. Sümen, Ö. Ö. (2023). Reflective thinking in the problem-solving process: A model proposal. Sakarya University Journal of Education, 13(1), 6-23. https://dergipark.org.tr/en/pub/suje/article/970213
62. Supriadi, N., & Suherman, S. (2024). The role of learning anxiety and mathematical reasoning as predictor of promoting learning motivation: The mediating role of mathematical problem solving. Thinking Skills and Creativity, 52, 101497. https://www.sciencedirect.com/science/article/pii/S187118712400035X
63. Tañola, M. D., & Lomibao, L. S. (2024). Understanding How Students Learn Mathematics: A systematic literature review of contemporary learning strategies in Mathematics education post-2020. Journal of Innovations in Teaching and Learning, 4(1), 66-75. https://www.researchgate.net/profile/Marivic-Tanola/publication/387602695_Understanding_How_Students_Learn_Mathematics_A_Systematic_Literature_Review_of_Contemporary_Learning_Strategies_in_Mathematics_Education_Post-2020/links/67754cce117f340ec3ea7aa6/Understanding-How-Students-Learn-Mathematics-A-Systematic-Literature-Review-of-Contemporary-Learning-Strategies-in-Mathematics-Education-Post-2020.pdf
64. Teng, L. S. (2022). Explicit strategy-based instruction in L2 writing contexts: A perspective of self-regulated learning and formative assessment. Assessing Writing, 53, 100645. https://www.sciencedirect.com/science/article/abs/pii/S1075293522000411
65. Thoe, N. K., Jamaludin, J., Pang, Y. J., Choong, C., Lay, Y. F., Ong, E. T., ... & Chin, C. K. (2022). Developing Conceptual and Procedural Knowledge/Skills of Lifelong Learners from Basic to Advance Learning: Exemplars, Challenges and Future Direction. Dinamika Jurnal Ilmiah Pendidikan Dasar, 14(1), 22-35. https://jurnalnasional.ump.ac.id/index.php/Dinamika/article/view/13164
66. Torres-Peña, R. C., Peña-González, D., Lara-Orozco, J. L., Ariza, E. A., & Vergara, D. (2025). Enhancing numerical thinking through problem solving: A teaching experience for third-grade mathematics. Education Sciences, 15(6), 667. https://www.mdpi.com/2227-7102/15/6/667
67. Ukobizaba, F., Nizeyimana, G., & Mukuka, A. (2021). Assessment Strategies for Enhancing Students' Mathematical Problem-Solving Skills: A Review of Literature. Eurasia Journal of Mathematics, Science and Technology Education, 17(3). https://eric.ed.gov/?id=EJ1289246
68. Vale, I., & Barbosa, A. (2023). Active learning strategies for an effective mathematics teaching and learning. European Journal of Science and Mathematics Education, 11(3), 573-588. https://www.scimath.net/article/active-learning-strategies-for-an-effective-mathematics-teaching-and-learning-13135
69. Vanderheyden, A. M., & Solomon, B. G. (2023). Valid outcomes for screening and progress monitoring: Fluency is superior to accuracy in curriculum-based measurement. School Psychology, 38(3), 160. https://psycnet.apa.org/record/2023-72860-003
70. Verschaffel, L. (2024). Strategy flexibility in mathematics. ZDM–Mathematics Education, 56(1), 115–126. https://tinyurl.com/5c8b5m4
71. Vettriselvan, R., Rajesh, D., Subhashini, S., Gajalakshmi, K., & Sakthivel, R. (2025). Developing and applying PCK in diverse subjects: Best practices for mathematics, science, social sciences, and language arts. In Current trends and best practices of pedagogical content knowledge (PCK) (pp. 1-30). IGI Global Scientific Publishing. https://www.igi-global.com/chapter/developing-and-applying-pck-in-diverse-subjects/369707
72. Yassin, A., & Mabanja, A. (2024, August). Integrating information processing theory with artificial intelligence for enhanced learning outcomes. In International conference on aplied social sciences in education (Vol. 1, No. 1, pp. 1-13). https://journal.upy.ac.id/index.php/icasse/article/view/6902
73. Zhang, J., Andres, J. M. A. L., Hutt, S., Baker, R. S., Ocumpaugh, J., Nasiar, N., & Young, T. (2022). Using machine learning to detect SMART model cognitive operations in mathematical problem-solving process. Journal of Educational Data Mining, 14(3), 76-108. https://jedm.educationaldatamining.org/index.php/JEDM/article/view/610