# Efficiency of Estimator of Probability Proportional to Size with and without Replacement

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**DOI: https://doi.org/10.51244/IJRSI.2026.1303000167**

**Received: 12 March 2026; Accepted: 17 March 2026; Published: 11 April 2026**

## ABSTRACT

Efficiency describes how well a sampling method extracts information from a population relative to effort or cost and a method is efficient if it gives precise estimate with a small sample. An estimator is efficient if it has a low variance. This research focused on the efficiency of probability proportional to size sampling scheme estimators. Probability proportional to size with replacement, Hansen-Hurwitz estimator and Probability proportional to size without replacement, Rao-Hartley-Cochran estimator were compared in terms of estimation and efficiency. The empirical comparison of the population total, minimum variance and relative efficiency were used in assessing the efficiency. Probability proportional to size with replacement, (Hansen-Hurwitz) estimator had the higher population total estimate for some years indicating that population total differed by year. Probability proportional to size sampling without replacement estimator, ( Rao–Hartley–Cochran) estimator thus had a smaller variance, hence more efficient than the conventional Hansen–Hurwitz estimator in terms of variance reduction.

**Keywords:** Probability; Hansen-Hurwitz Estimator; Rao–Hartley–Cochran Estimator; Population Total; Minimum Variance; Relative Efficiency

## INTRODUCTION

Sampling is used extensively in research in many different domains. Due to time, cost, and accessibility limitations, it is frequently impracticable to gather data from a whole population (Cochran, 1977). Rather, in order to accurately estimate population totals and other characteristics, researchers use sampling procedures to choose a representative subset of the population. The effectiveness of these estimators is crucial because of precision and dependability of statistical findings (Lohr, 2021). A number of estimators have been proposed by numerous researchers by effectively modifying the auxiliary variables under Probability proportional to Size (PPS). For microscope applications, Abdulla et al. (2014) and Andersen et al. (2015) suggested optimal PPS sampling with vanishing auxiliary variables. Alam et al. (2015) suggested using a probability proportion to size when choosing samples. Assigning distinct selection probabilities to various population units allows one to calculate the probability proportional to size (Pamplona, 2019). Selection probabilities can therefore be allocated proportionately to population unit size when there is a significant variation in unit size and a strong correlation between variance and unit size. Because auxiliary information is used, it is argued that the essence of probability proportional to size is preferable to that of unbiased sampling procedures. The emphasis on the necessity and use of supplementary information to improve estimate precision led to advancements in sampling theory with the introduction of probability proportional to size estimators (Homa, et al, 2013). Probability proportional to size can be operated on the basis of procedures involving both replacement and non-replacement, just like any other equal sampling scheme. The standard is the conventional probability proportional to size, with replacement. Horvitz-Thompson (1952) made the first adjustment, suggesting a probability proportional to size without replacement. Johnson et al (2015) assessed effectiveness of probability proportional to size with replacement (PPSWR) in calculating national household expenditures and found that, PPSWR produced estimates that were more stable in groups with wildly disparate wealth distributions. They showed that auxiliary information increased estimate accuracy, which made it PPSWR appropriate for extensive economic analyses. The effectiveness of the Rao-Hartley-Cochran (RHC) approach in measuring unemployment gaps across various demographic groups was examined by Hughes and Bennett (2025), who found that RHC decreased estimation errors in populations with high worker mobility, and concluded that RHC should be used in economic research to inform workforce training initiatives and employment regulations.

# METHODOLOGY

**Probability Proportional to Size with replacement (Hansen and Hurwitz Estimator)**

Hansen and Hurwitz (1943) developed the general theory of probability proportional to size with replacement. One unit was selected at each of the n draws. They allocated the selection probability to the $i^{th}$ unit of the population given by

$$P_i = \frac{Y_i}{Y} \qquad 2.1$$

where $Y_i$ (auxiliary variable) measure the of size of the $i^{th}$ population unit and $Y = \sum_{i=1}^{N} Y_i$.

**Unbiased estimate of Population total**

If a sample of size n units is drawn with PPS of $x_i$ and with replacement, then

$$\hat{Y}_{HH} = \frac{1}{n}\sum_{i}^{n}\frac{y_i}{p_i} \qquad 2.2$$

is an unbiased estimate of the population total Y. where $p_i = \frac{x_i}{x}$ is the probability of selecting the ith unit in the sample.

**Unbiased estimator of Variance of Hansen Hurwitz estimator**

If a sample of n units is drawn with PPS of $x_i$ and with replacement, an unbiased sample estimate of $V(\hat{Y}_{HH})$ is, for any $n > 1$,

$$v(\hat{Y}_{HH}) = \sum_{i}^{n}\frac{(y_i/p_i - \hat{Y}_{HH})^2}{n(n-1)} \qquad 2.3$$

**Probability Proportional to Size Sampling Without Replacement (PPSWOR):**

**Rao-Hartley-Cochran Estimator**

Rao et al. (1962) proposed a sampling strategy for use with unequal probability sampling and the estimator of population total. The population units are divided randomly into n groups, where the group sizes are predetermined. Then one unit is selected from each group.

**Unbiased estimator of Rao Hartley Cochran estimator**

The unbiased estimator of population total Y is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n}\frac{y_{il}}{(P_{il}/\tau_i)} \qquad 2.4$$

**Unbiased estimator of the Variance of Rao Hartley Cochran estimator**

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue III March 2026

An unbiased estimator of the variance $V(\hat{Y}_{RHC})$ of Rao Hartley Cochran is given by

$$\hat{v}(\hat{Y}_{RHC}) = \frac{\left(\sum_{i=1}^{n} N_i^2 - N\right)}{\left(N^2 - \sum_{i=1}^{n} N_i^2\right)} \left[\sum_{i=1}^{n} \frac{y_{i1}^2}{(P_{i1}^2/\tau_i)} - \hat{Y}_{RHC}^2\right]. \qquad 2.5$$

**Mathematical Comparison of PPSWR with PPSWOR**

The Rao Hartley Cochran (RHC) scheme is more efficient than PPSWR sampling

i.e. $V(\hat{Y}_{RHC}) < V(\hat{Y}_{PPSWR})$ if $N_i = \frac{N}{n}, \forall\, i = 1, 2, \ldots, n.$

**Empirical Comparison of PPSWR with PPSWOR**

This study was used to compare the efficiency of sampling scheme estimators in estimating the population total of all registered live births (double births) by type of registration center across all Local Government Areas (LGAs) in Ekiti State. One PPSWR estimator (Hansen Hurwitz) and one PPSWOR estimator (Rao-Hartley-Cochran) with regards to population total and variations in live births for the three-year period 2022–2024 was presented. Thus, the outcomes are shown below:

**Probability Proportional to Size Sampling Scheme**

**Range and Selection of Samples among the LGAs with cumulative total**

Table 3.1: Range and Selection of Samples among the LGAs

| S/N | LGA | No of Reg. Centre | Cum Total | Ranges | Total Birth (2022) | Total Birth (2023) | Total Birth (2024) | Total Double Birth (2022) | Total Double Birth (2023) | Total Double Birth (2024) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Ado* | 11 | 11 | 1 – 11 | 10419 | 9553 | 10800 | 331 | 286 | 410 |
| 2 | Efon | 4 | 15 | 12 – 15 | 1470 | 1689 | 1532 | 23 | 30 | 24 |
| 3 | Ekiti East* | 11 | 26 | 16 – 26 | 2086 | 3618 | 1918 | 77 | 116 | 72 |
| 4 | Ekiti South West | 4 | 30 | 27 – 30 | 1112 | 1360 | 1106 | 37 | 70 | 46 |
| 5 | Ekiti West | 5 | 35 | 31 – 35 | 1461 | 1644 | 1422 | 41 | 64 | 30 |
| 6 | Emure | 4 | 39 | 36 – 39 | 902 | 1261 | 849 | 29 | 46 | 34 |
| 7 | Gbonyin* | 6 | 45 | 40 – 45 | 1710 | 1917 | 1611 | 36 | 59 | 36 |
| 8 | Ido Osi | 4 | 49 | 46 – 49 | 2190 | 2173 | 1730 | 50 | 60 | 56 |
| 9 | Ijero | 6 | 55 | 50 – 55 | 2244 | 2342 | 2602 | 80 | 133 | 96 |
| 10 | Ikere* | 5 | 60 | 56 – 60 | 1866 | 1994 | 1953 | 62 | 128 | 14 |

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| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | Ikole | 7 | 67 | 61 – 67 | 2106 | 2202 | 1783 | 49 | 60 | 16 |
| 12 | Ilejemeje | 3 | 70 | 68 – 70 | 786 | 789 | 814 | 32 | 35 | 15 |
| 13 | Irepodun/ Ifelodun | 6 | 76 | 71 – 76 | 2348 | 2430 | 1897 | 55 | 64 | 70 |
| 14 | Ise Orun* | 4 | 80 | 77 – 80 | 1601 | 1553 | 1145 | 34 | 36 | 22 |
| 15 | Moba | 6 | 86 | 81 – 86 | 1620 | 1692 | 1500 | 59 | 108 | 63 |
| 16 | Oye | 4 | 90 | 87 – 90 | 1753 | 1677 | 1610 | 44 | 50 | 41 |

**Note:** 5 LGAs were selected based on random sampling with replacement

**Source:** National Population Commission, Ado Ekiti

**Estimation of Total Population**

An unbiased estimator of the population total under probability proportional to size with replacement, Y is given by

$$\hat{Y}_{PPS} = \frac{1}{n}\sum_{i=1}^{N} {y_i}/{p_i} \qquad 2.6$$

Table 3. 2: Probabilities and Yearly data of child's birth for the sampled LGAs

| **LGAs** | **Ado** | **Ekiti East** | **Gbonyin** | **Ikere** | **Ise Orun** |
|---|---|---|---|---|---|
| $\boldsymbol{p_i}$ | ${11}/{90}$= 0.1222 | ${11}/{90}$ = 0.1222 | ${6}/{90}$ = 0.0667 | ${5}/{90}$ = 0.0555 | ${4}/{90}$= 0.0444 |
| $\boldsymbol{y_{i(2022)}}$ | 331 | 77 | 36 | 62 | 34 |
| $\boldsymbol{y_{i(2023)}}$ | 286 | 116 | 59 | 128 | 36 |
| $\boldsymbol{y_{i(2024)}}$ | 410 | 72 | 36 | 14 | 22 |

**Total Population Estimate for 2022**

$$\hat{Y}_{PPS} = \frac{90}{5}\left[\frac{331}{11} + \frac{77}{11} + \frac{36}{6} + \frac{62}{5} + \frac{34}{4}\right]$$

$$\hat{Y}_{PPS} = 18(30.09 + 7 + 6 + 12.4 + 8.5)$$

$$\hat{Y}_{PPS} = 18(63.99) = 1151.82 \sim 1152\ double\ births$$

**Total Population Estimate for 2023**

$$\hat{Y}_{PPS} = \frac{90}{5}\left[\frac{286}{11} + \frac{116}{11} + \frac{59}{6} + \frac{128}{5} + \frac{36}{4}\right]$$

$$\hat{Y}_{PPS} = 18(26 + 10.55 + 9.83 + 25.6 + 9)$$

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$$\hat{Y}_{PPS} = 18(80.98) = 1457.64 \sim 1458 \; double \; births$$

**Total Population Estimate for 2024**

$$\hat{Y}_{PPS} = \frac{90}{5}\left[\frac{410}{11} + \frac{72}{11} + \frac{36}{6} + \frac{14}{5} + \frac{22}{4}\right]$$

$$\hat{Y}_{PPS} = 18(37.27 + 6.55 + 6 + 2.8 + 5.5)$$

$$\hat{Y}_{PPS} = 18(58.12) = 1046.16 \sim 1046 \; double \; births$$

**Estimation of Variance**

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left({}^{y_i}/_{p_i} - \hat{Y}_{pps}\right)^2 \qquad 2.7$$

**Variance Estimate for 2022**

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[\left(\frac{331}{0.1222}\right)^2 + \left(\frac{77}{0.1222}\right)^2 + \left(\frac{36}{0.0667}\right)^2 + \left(\frac{62}{0.0555}\right)^2 + \left(\frac{34}{0.0444}\right)^2 - 5(1151.82)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(2708.67)^2 + (630.11)^2 + (539.73)^2 + (1117.12)^2 + (765.77)^2 - 5(1151.82)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(7336916.49 + 397044.37 + 291308.62 + 1247950.65 + 586397.21) - 5(1326689.31)\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(9859617.33 - 6633446.56)$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(3226170.77)$$

$$\hat{V}(\hat{Y}_{pps}) = 161308.54$$

Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{161308.54}$$

$$SE(\hat{Y}_{pps}) = 401.63$$

**Variance Estimate for 2023**

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[\left(\frac{286}{0.1222}\right)^2 + \left(\frac{116}{0.1222}\right)^2 + \left(\frac{59}{0.0667}\right)^2 + \left(\frac{128}{0.0555}\right)^2 + \left(\frac{36}{0.0444}\right)^2 - 5(1457.64)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(2340.43)^2 + (949.26)^2 + (884.56)^2 + (2306.31)^2 + (810.81)^2 - 5(1457.64)^2\right]$$

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$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(5477591.67 + 901101.20 + 782442.36 + 5319048.78 + 657414.17) - 5(2124714.37)\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(13137598.18 - 10623571.85)$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(2514026.33)$$

$$\hat{V}(\hat{Y}_{pps}) = 125701.32$$

Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{125701.32}$$

$$SE(\hat{Y}_{pps}) = 354.54$$

**Variance Estimate for 2024**

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[\left(\frac{410}{0.1222}\right)^2 + \left(\frac{72}{0.1222}\right)^2 + \left(\frac{36}{0.0667}\right)^2 + \left(\frac{14}{0.0555}\right)^2 + \left(\frac{22}{0.0444}\right)^2 - 5(1046.16)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(3355.16)^2 + (589.20)^2 + (539.73)^2 + (252.25)^2 + (495.50)^2 - 5(1046.16)^2\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{5(4)}\left[(11257068.31 + 347154.33 + 291308.62 + 63631.20 + 245515.79) - 5(1094450.75)\right]$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(12204678.24 - 5472253.73)$$

$$\hat{V}(\hat{Y}_{pps}) = \frac{1}{20}(6732424.51)$$

$$\hat{V}(\hat{Y}_{pps}) = 336621.23$$

Standard Error (SE) is

$$SE(\hat{Y}_{pps}) = \sqrt{336621.23}$$

$$SE(\hat{Y}_{pps}) = 580.19$$

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**Rao-Hartley-Cochran's (RHC) Sampling Scheme**

Table 3. 3: Total Registered Live Births by Type of Birth by Registration Centre Across All LGA in Ekiti State for 2022, 2023 & 2024

| S/N | LGA | No of Registration Centres | Total Double Birth (2022) | Total Double Birth (2023) | Total Double Birth (2024) |
|---|---|---|---|---|---|
| 1 | Ado | 11 | 331 | 286 | 410 |
| 2 | Efon | 4 | 23 | 30 | 24 |
| 3 | Ekiti East | 11 | 77 | 116 | 72 |
| 4 | Ekiti South West | 4 | 37 | 70 | 46 |
| 5 | Ekiti West | 5 | 41 | 64 | 30 |
| 6 | Emure | 4 | 29 | 46 | 34 |
| 7 | Gbonyin | 6 | 36 | 59 | 36 |
| 8 | Ido Osi | 4 | 50 | 60 | 56 |
| 9 | Ijero | 6 | 80 | 133 | 96 |
| 10 | Ikere | 5 | 62 | 128 | 14 |
| 11 | Ikole | 7 | 49 | 60 | 16 |
| 12 | Ilejemeje | 3 | 32 | 35 | 15 |
| 13 | Irepodun/ Ifelodun | 6 | 55 | 64 | 70 |
| 14 | Ise Orun | 4 | 34 | 36 | 22 |
| 15 | Moba | 6 | 59 | 108 | 63 |
| 16 | Oye | 4 | 44 | 50 | 41 |
| | **Total** | | 1039 | 1345 | 1045 |

Following the numbers of Local Government Areas in the population collected above, we are to select a sample of size 4 using RHC scheme. Thus, the population (LGAs) will be divided into four (4) random groups. To do this, we selected 16 distinct random numbers between 1 and 16 using the Random Numbers Table in the following sequence; 10, 11, 01, 02, 08, 16, 05, 12, 09, 13, 04, 14, 06, 15, 03, 07.

The LGAs bearing the serial numbers corresponding to the first four selected random numbers constitute the first random group, whereas, the next four random numbers form the second random group and so on.

Let $X_{ij} = Numbers\ of\ Registration\ centres\ \ for\ the\ jth\ LGA\ in\ the\ ith\ random\ group$

$$Y_{ij} = Registration\ of\ double\ birthfor\ the\ jth\ LGA\ in\ the\ ith\ random\ group$$

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$$P_{ij} = The\ initial\ selection\ probability\ of\ the\ jth\ unit\ in\ the\ ith\ random\ group$$

$$\sum Y_{ij\,(2022)} = 1039, \sum Y_{ij\,(2023)} = 1345, \sum Y_{ij\,(2024)} = 1045, \sum X_{ij} = 90$$

Therefore, the following are the 4 random groups of units along with the initial selection of probability.

Table 3. 4: first random group by RHC selection

| S/N | RANDOM NO. | LGA | $X_{ij}$ | $Y_{ij(2022)}$ | $Y_{ij(2023)}$ | $Y_{ij(2024)}$ | $P_{ij}$ |
|---|---|---|---|---|---|---|---|
| 1 | 10 | Ikere | 5 | 62 | 128 | 14 | 0.0556 |
| 2 | 11 | Ikole | 7 | 49 | 60 | 16 | 0.0778 |
| 3 | 01 | Ado | 11 | 331 | 286 | 410 | 0.1222 |
| 4 | 02 | Efon | 4 | 23 | 30 | 24 | 0.0444 |
| | | **Total** | 27 | **465** | **504** | **464** | **0.3000** |

Table 3. 5: Second random group by RHC selection

| S/N | RANDOM NO. | LGA | $X_{ij}$ | $Y_{ij(2022)}$ | $Y_{ij(2023)}$ | $Y_{ij(2024)}$ | $P_{ij}$ |
|---|---|---|---|---|---|---|---|
| 1 | 08 | Ido/Osi | 4 | 50 | 60 | 56 | 0.0444 |
| 2 | 16 | Oye | 4 | 44 | 50 | 41 | 0.0444 |
| 3 | 05 | Ekiti West | 5 | 41 | 64 | 30 | 0.0556 |
| 4 | 12 | Ilejemeje | 3 | 32 | 35 | 15 | 0.0333 |
| | | **Total** | 16 | **167** | **209** | **142** | **0.1778** |

Table 3. 6: Third random group by RHC selection

| S/N | RANDOM NO. | LGA | $X_{ij}$ | $Y_{ij(2022)}$ | $Y_{ij(2023)}$ | $Y_{ij(2024)}$ | $P_{ij}$ |
|---|---|---|---|---|---|---|---|
| 1 | 09 | Ijero | 6 | 80 | 133 | 96 | 0.0667 |
| 2 | 13 | Irepodun/ Ifelodun | 6 | 55 | 64 | 70 | 0.0667 |
| 3 | 04 | Ekiti South West | 4 | 37 | 70 | 46 | 0.0444 |
| 4 | 14 | Ise/Orun | 4 | 34 | 36 | 22 | 0.0444 |
| | | **Total** | 20 | **206** | **303** | **234** | **0.2222** |

Table 3. 7: Fourth random group by RHC selection

| S/N | RANDOM NO. | LGA | $X_{ij}$ | $Y_{ij(2022)}$ | $Y_{ij(2023)}$ | $Y_{ij(2024)}$ | $P_{ij}$ |
|---|---|---|---|---|---|---|---|
| 1 | 06 | Emure | 4 | 29 | 46 | 34 | 0.0444 |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 15 | Moba | 6 | 59 | 108 | 63 | 0.0667 |
| 3 | 03 | Ekiti East | 11 | 77 | 116 | 72 | 0.1222 |
| 4 | 07 | Gbonyin | 6 | 36 | 59 | 36 | 0.0667 |
| | | **Total** | 27 | **201** | **329** | **205** | **0.3000** |

Units are thus selected independently in each group using a method of selection (Lahiri's method).

The first random group consists of $N_1 = 4$ units and maximum value of the variable $X_{1j} = 11$. Choosing $X_0 = 15$, we select random number $1 \leq R_i \leq 4$ by starting with 4th and 5th columns and another random number $1 \leq R_j \leq 15$ by starting from 23rd and 24th columns of the Random Number Table. Then the first effective pair of random number is (04, 05) as shown in the table below. Thus, from the first random group, Ikere LGA was included in the sample.

Table 3. 8: selected LGA for group 1 by RHC selection

| Trial No. | Group S/N ($1 \leq R_i \leq N$) | LGA | $1 \leq R_j \leq N$ | $X_{1j}$ | Decision R = Rejected S = Selected |
|---|---|---|---|---|---|
| 1 | 04 | Efon | 05 | 4 | R |
| 2 | 03 | Ado | 14 | 11 | R |
| 3 | 02 | Ikole | 12 | 7 | R |
| 4 | 01 | Ikere | 04 | 5 | S |

The second random group consists of $N_2 = 4$ units and maximum value of the variable $X_{2j} = 5$. Choosing $X_0 = 10$, we select random number $1 \leq R_i \leq 4$ by starting with 13th and 14th columns and another random number $1 \leq R_j \leq 10$ by starting from 22nd to 24th columns of the Random Number Table. Then the first effective pair of random number is (02, 04) as shown in the table below. Thus, from the first random group, Oye LGA was included in the sample.

Table 3.9: Selected LGA for group 2 by RHC selection

| Trial No. | Group S/N ($1 \leq R_i \leq N$) | LGA | $1 \leq R_j \leq N$ | $X_{2j}$ | Decision R = Rejected S = Selected |
|---|---|---|---|---|---|
| 1 | 01 | Ido/Osi | 09 | 4 | R |
| 2 | 04 | Ilejemeje | 07 | 3 | R |
| 3 | 02 | Oye | 02 | 4 | S |
| 4 | 03 | Ekiti West | 06 | 5 | R |

The third random group consists of $N_3 = 4$ units and maximum value of the variable $X_{3j} = 6$. Choosing $X_0 = 10$, we select random number $1 \leq R_i \leq 4$ by starting with 19th and 20th columns and another random number $1 \leq R_j \leq 10$ by starting from 15th and 16th columns of the Random Number Table. Then the first effective pair of random number is (03, 01) as shown in the table below. Thus, from the first random group, Ekiti South West LGA was included in the sample.

Table 3.10: Selected LGA for group 3 by RHC selection

| Trial No. | Group S/N ($1 \leq R_i \leq N$) | LGA | $1 \leq R_j \leq N$ | $X_{3j}$ | Decision R = Rejected S = Selected |
|---|---|---|---|---|---|
| 1 | 03 | Ekiti South West | 01 | 4 | S |
| 2 | 02 | Irepodun/ Ifelodun | 08 | 6 | R |
| 3 | 01 | Ijero | 10 | 6 | R |
| 4 | 04 | Ise/ Orun | 06 | 4 | R |

The fourth random group consists of $N_4 = 4$ units and maximum value of the variable $X_{4j} = 11$. Choosing $X_0 = 15$, we select random number $1 \leq R_i \leq 4$ by starting with 41st and 42nd columns and another random number $1 \leq R_j \leq 15$ by starting from 31st to 32nd columns of the Random Number Table. Then the first effective pair of random number is (03, 11) as shown in the table below. Thus, from the first random group, Ekiti East LGA will be included in the sample.

Table 3. 11: selected LGA for group 4 by RHC selection

| Trial No. | Group S/N ($1 \leq R_i \leq N$) | LGA | $1 \leq R_j \leq N$ | $X_{4j}$ | Decision R = Rejected S = Selected |
|---|---|---|---|---|---|
| 1 | 03 | Ekiti East | 07 | 11 | S |
| 2 | 02 | Moba | 04 | 6 | S |
| 3 | 04 | Gbonyin | 14 | 6 | R |
| 4 | 01 | Emure | 10 | 4 | R |

Results presented in Table 3.8 – Table 3.11 revealed that there are four random groups for the Local Government Areas covered in the study. For the first random group, there are Ikere, Ikole, Ado and Efon. In the second random group we have Ido/Osi, Oye, Ekiti West and Ilejemeje. The third random group consists of Ijero, Irepodun/Ifelodun, Ekiti South West, Ise/Orun while the last random group is made up of Emure, Moba, Ekiti East and Gbonyin. In line with selection of one LGA from each of the groups, based on random sampling and Lahiri's selection method, the LGAs selected were Ikere, Oye, Ekiti South West and Ekiti East for first, second, third and fourth group respectively.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XIII Issue III March 2026

**Probability of Selection under Rao-Hartley-Cochran's estimator**

Table 3.12: Sampled LGAs and Selection Probabilities for double birth in 2022

| S/N | LGA | $Y_{i1}$ | $X_{i1}$ | $P_{i1}$ | $\tau_i$ | $\frac{P_{i1}}{\tau_i}$ | $\frac{Y_{i1}}{(P_{i1}/\tau_i)}$ | ${Y_{i1}}^2$ | ${P_{i1}}^2$ | $\frac{{P_{i1}}^2}{\tau_i}$ | $\frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Ikere | 5 | 62 | 0.0556 | 0.3000 | 0.1852 | 334.80 | 3844 | 0.0031 | 0.0103 | 373636.80 |
| 2 | Oye | 4 | 44 | 0.0444 | 0.1778 | 0.2500 | 176.02 | 1936 | 0.0020 | 0.0111 | 174261.78 |
| 3 | Ekiti South West | 4 | 37 | 0.0444 | 0.2222 | 0.2000 | 184.98 | 1369 | 0.0020 | 0.0089 | 153997.10 |
| 4 | Ekiti East | 11 | 77 | 0.1222 | 0.3000 | 0.4074 | 189.00 | 5929 | 0.0149 | 0.0498 | 119070.00 |
| | **Total** | | | | | | **884.80** | | | | **820965.68** |

Note that

$$\tau_i = \sum_{j\in G_i} P_{ij}\,, \qquad i = 1,2,3,4$$

Population Total for RHC is given as

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{\boldsymbol{Y_{i1}}}{(\boldsymbol{P_{i1}}/\boldsymbol{\tau_i})}$$

The Total population estimate of double births for the year 2022 is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{\boldsymbol{Y_{i1}}}{(\boldsymbol{P_{i1}}/\boldsymbol{\tau_i})} = \boldsymbol{884.80}$$

An estimate of variance of the estimator $\hat{Y}_{RHC}$ is given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{(N^2 - \sum_{i=1}^{n} N_i^2)} \left[ \sum_{i=1}^{n} \frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i} - \hat{Y}_{RHC}^2 \right]$$

$$= \frac{(\sum_{i=1}^{4} 4^2 - 16)}{(16^2 - \sum_{i=1}^{4} 4^2)} [820965.68 - (884.80)^2]$$

$$= \frac{[(16+16+16+16) - (16)]}{[(256) - (16+16+16+16)]} [820965.68 - 782877.23]$$

$$= \frac{64}{192} [38088.45]$$

$$= 12696.15$$

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Table 3.13: Sampled LGAs and Selection Probabilities for double birth in 2023

| S/N | LGA | $Y_{i1}$ | $X_{i1}$ | $P_{i1}$ | $\tau_i$ | $\frac{P_{i1}}{\tau_i}$ | $\frac{Y_{i1}}{(P_{i1}/\tau_i)}$ | ${Y_{i1}}^2$ | ${P_{i1}}^2$ | $\frac{{P_{i1}}^2}{\tau_i}$ | $\frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Ikere | 5 | 128 | 0.0556 | 0.3000 | 0.1852 | 691.20 | 16384 | 0.0031 | 0.01028807 | 1592524.8 |
| 2 | Oye | 4 | 50 | 0.0444 | 0.1778 | 0.2500 | 200.03 | 2500 | 0.0020 | 0.01110972 | 225028.125 |
| 3 | Ekiti South West | 4 | 70 | 0.0444 | 0.2222 | 0.2000 | 349.97 | 4900 | 0.0020 | 0.00888978 | 551194.875 |
| 4 | Ekiti East | 11 | 116 | 0.1222 | 0.3000 | 0.4074 | 284.73 | 13456 | 0.0149 | 0.04979424 | 270232.0661 |
| | **Total** | | | | | | **1525.92** | | | | **2638979.87** |

Note that

$$\tau_i = \sum_{j \in G_i} P_{ij}, \qquad i = 1, 2, 3, 4$$

The Total population estimate of double births for the year 2023 is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{Y_{i1}}{(P_{i1}/\tau_i)} = \mathbf{1525.92}$$

An estimate of variance of the estimator $\hat{Y}_{RHC}$ is given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{(N^2 - \sum_{i=1}^{n} N_i^2)} \left[ \sum_{i=1}^{n} \frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i} - \hat{Y}_{RHC}^2 \right]$$

$$= \frac{(\sum_{i=1}^{4} 4^2 - 16)}{(16^2 - \sum_{i=1}^{4} 4^2)} [2638979.87 - (1525.92)^2]$$

$$= \frac{[(16 + 16 + 16 + 16) - (16)]}{[(256) - (16 + 16 + 16 + 16)]} [2638979.87 - 2328423.52]$$

$$= \frac{64}{192} [310556.34]$$

$$= 103518.78$$

Table 3.14: Sampled LGAs and Selection Probabilities for double birth in 2024

| S/N | LGA | $Y_{i1}$ | $X_{i1}$ | $P_{i1}$ | $\tau_i$ | $\frac{P_{i1}}{\tau_i}$ | $\frac{Y_{i1}}{(P_{i1}/\tau_i)}$ | ${Y_{i1}}^2$ | ${P_{i1}}^2$ | $\frac{{P_{i1}}^2}{\tau_i}$ | $\frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Ikere | 5 | 14 | 0.0556 | 0.3000 | 0.1852 | 75.60 | 196 | 0.0031 | 0.01028807 | 19051.2 |

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| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | Oye | 4 | 41 | 0.0444 | 0.1778 | 0.2500 | 164.02 | 1681 | 0.0020 | 0.01110972 | 151308.9113 |
| 3 | Ekiti South West | 4 | 46 | 0.0444 | 0.2222 | 0.2000 | 229.98 | 2116 | 0.0020 | 0.00888978 | 238026.195 |
| 4 | Ekiti East | 11 | 72 | 0.1222 | 0.3000 | 0.4074 | 176.73 | 5184 | 0.0149 | 0.04979424 | 104108.4298 |
| | **Total** | | | | | | **646.32** | | | | **512494.74** |

Note that

$$\boldsymbol{\tau_i = \sum_{j \in G_i} P_{ij}\,, \qquad i = 1, 2, 3, 4}$$

The Total population estimate of double births for the year 2024 is given by

$$\hat{Y}_{RHC} = \sum_{i=1}^{n} \frac{\boldsymbol{Y_{i1}}}{(\boldsymbol{P_{i1}}/\boldsymbol{\tau_i})} = \mathbf{646.32}$$

An estimate of variance of the estimator $\hat{Y}_{RHC}$ is given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i=1}^{n} N_i^2 - N)}{(N^2 - \sum_{i=1}^{n} N_i^2)} \left[ \sum_{i=1}^{n} \frac{{Y_{i1}}^2}{{P_{i1}}^2/\tau_i} - \hat{Y}_{RHC}^2 \right]$$

$$= \frac{(\sum_{i=1}^{4} 4^2 - 16)}{(16^2 - \sum_{i=1}^{4} 4^2)} [512494.74 - (646.32)^2]$$

$$= \frac{[(16 + 16 + 16 + 16) - (16)]}{[(256) - (16 + 16 + 16 + 16)]} [512494.74 - 417735.71]$$

$$= \frac{64}{192} [94759.02]$$

$$= 31586.34$$

**Sen – Yates – Grundy Sampling Scheme**

Table 3.15: Total Population Estimate for PPSWR and PPSWOR

| **Year** | $\hat{Y}_{PPSWR}$ | $\hat{Y}_{PPSWOR}$ |
|---|---|---|
| **2022** | 1152 | 884.80 |
| **2023** | 1458 | 1525.92 |
| **2024** | 1046 | **646.32** |

Bar Chart Presentations for PPSWR and PPSWOR Estimates (2022–2024)

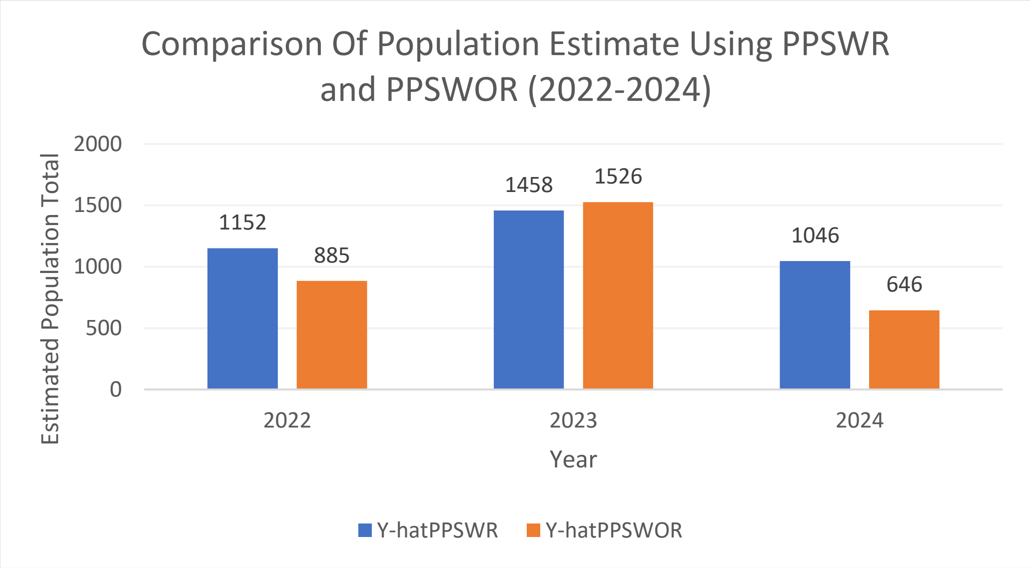


Fig 3. 1: Total Population Estimate for PPSWR and PPSWOR from 2022-2024

Table 3.3 and fig 3.1 revealed that the PPSWR estimator (Hansen Hurwitz) had the higher population total estimate for year 2022 and 2024 while the PPSWOR (Rao-Hartley-Cochran) had the higher population total estimate for year 2023 only.

Table 3:15 Population Variance Estimate for PPSWR and PPSWOR

| Year | $V(\hat{Y}_{PPSWR})$ | $V(\hat{Y}_{PPSWOR})$ |
|---|---|---|
| **2022** | 161308.54 | 12696.15 |
| **2023** | 125701.32 | 103518.78 |
| **2024** | 336621.23 | 31586.34 |

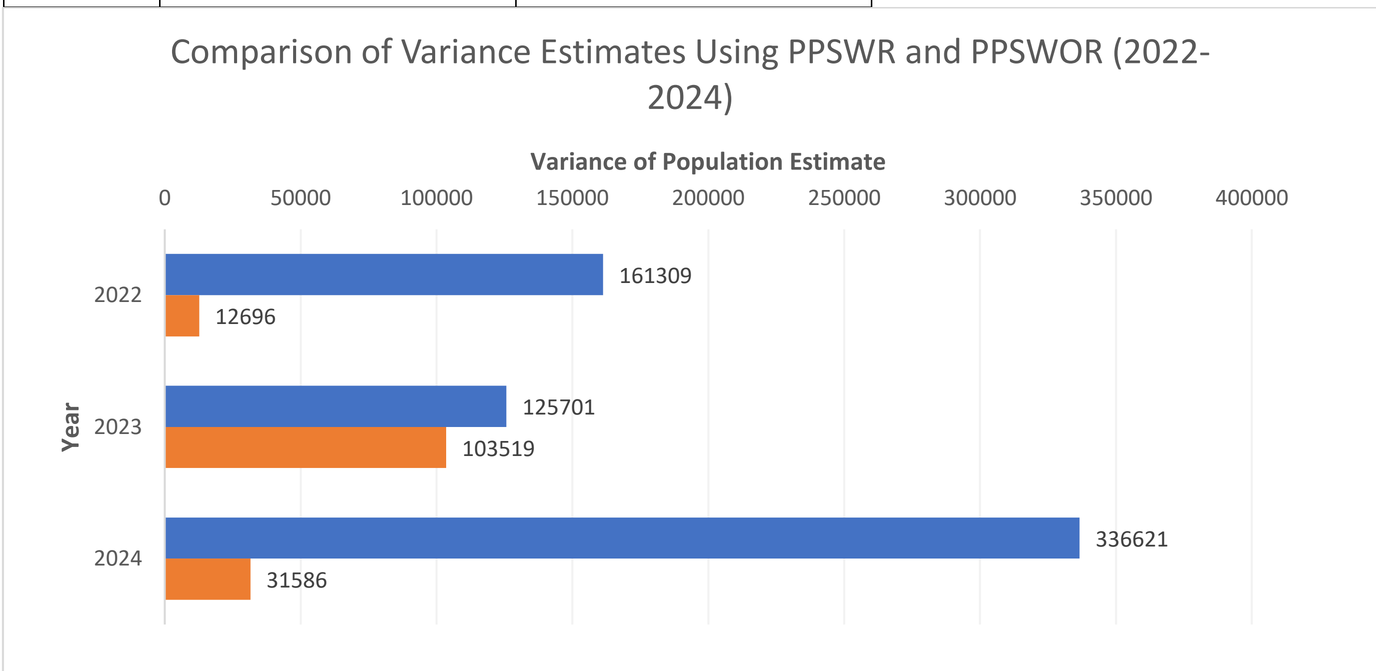


Fig 3. 2: Variance of Population Total

Table 3.4 and fig 3. 2 revealed that PPSWOR (Rao-Hartley-Cochran) had the lower variance across the years. This implies PPSWOR (Rao-Hartley-Cochran) is efficient than PPSWR estimator (Hansen Hurwitz).

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Table 3.16 Relative Efficiency of PPSWOR estimator to PPSWR estimator

| **Year** | **Relative Efficiency (%)** |
|---|---|
| 2022 | 1270.53 |
| 2023 | 121.43 |
| 2024 | 1065.72 |

PPSWOR (Rao-Hartley-Cochran) is approximately 1271% more efficient than PPSWR estimator (Hansen Hurwitz) at year 2022.

PPSWOR (Rao-Hartley-Cochran) is approximately 121% more efficient than PPSWR estimator (Hansen Hurwitz) at year 2023.

PPSWOR (Rao-Hartley-Cochran) is approximately 1066% more efficient than PPSWR estimator (Hansen Hurwitz) at year 2024.

## CONCLUSION

The findings from this research showed that the projected population total of any given population is greatly affected by the estimator used for the sampling technique. PPSWR estimator (Hansen Hurwitz) had the higher population total estimate for some years indicating that population total differed by year and the underlying data distribution, rather than any estimator being consistently better over all time periods. In terms of variance reduction, probability proportional to size sampling without replacement estimators specifically, Rao–Hartley–Cochran and Sen–Yates–Grundy—generally perform better than the conventional Hansen–Hurwitz estimator

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