# Multivariate Monitoring of Gross Domestic Product and Inflation Rate in Ghana

**Mutala Mohammed, Wahab Mashud, Abu Ibrahim Azebre**

**Department of Statistics and Actuarial Science, C.K. Tedam University of Technology and Applied Sciences**

**DOI: https://dx.doi.org/10.51244/IJRSI.2025.12110089**

**Received: 07 October 2025; Accepted: 12 October 2025; Published: 09 December 2025**

## ABSTRACT

Multivariate control charts are statistical tools increasingly used for the simultaneous monitoring of multiple interrelated variables. This study applied Hotelling $T^2$, multivariate cumulative sum (MCUSUM), and multivariate exponentially weighted moving average (MEWMA) control charts to jointly monitor Gross Domestic Product (GDP) and inflation rate in Ghana, aiming to detect both small and large shifts in the mean vector of these variables. Annual data for the period 1973–2022 were obtained from the Bank of Ghana. Results indicate that the Hotelling $T^2$ chart flagged out-of-control points in 1976, 1980, 1982, 2012, and 2013, primarily reflecting moderate-to-large shifts in GDP and inflation. The MCUSUM chart detected a deviation in 1982, while the MEWMA chart identified out-of-control points in 1980, 1982, and 2013, capturing subtle but persistent changes. Comparative analysis suggests that Hotelling $T^2$ is most effective for detecting moderate-to-large shifts, whereas MCUSUM and MEWMA provide complementary sensitivity to smaller or time-weighted changes. This study is novel in applying multivariate SPC techniques to Ghana's macroeconomic indicators, offering a proactive framework for monitoring GDP and inflation jointly. Integrating such charts into the Bank of Ghana's economic monitoring tools could facilitate earlier detection of macroeconomic deviations and support more informed policy responses.

## BACKGROUND

Macroeconomic and financial statistics are fundamental to shaping national economic policies, as they provide insights into a country's wealth, economic performance, and financial health through indicators such as gross domestic product (GDP) and inflation. GDP, defined under the System of National Accounts 2008 as the total market value of goods and services produced within a specific period, remains central to budget planning and economic forecasting (United Nations et al., 2009; Bade, 2016). It is widely regarded as the most reliable measure of a nation's economic health because it captures both the scale and dynamics of production.

Inflation, understood as a sustained rise in the general price level, reduces purchasing power and distorts decision-making by households, firms, and policymakers. It may be driven by factors such as fiscal imbalances, monetary expansion, or surges in external demand. According to the International Monetary Fund, low, stable, and predictable inflation is critical for fostering long-term growth and macroeconomic stability (Oner, 2010). Economists typically distinguish among creeping, walking, galloping, and hyperinflation, each reflecting different magnitudes of price acceleration and distinct policy challenges.

In Ghana, monetary policy is entrusted to the Bank of Ghana, which operates an inflation-targeting framework with the objective of ensuring price stability as a foundation for sustainable growth (Abradu-Otoo et al., 2024). Although maintaining stable inflation is a critical goal (Alhassan & Fiador, 2014), it does not by itself provide a full picture of overall economic health, highlighting the need for complementary indicators such as GDP to capture broader macroeconomic performance. This policy orientation underscores the centrality of inflation control in Ghana's macroeconomic management, yet it also raises the question of how price dynamics interact with output performance, suggesting the importance of studying GDP and inflation together rather than in isolation.

Previous studies have mainly analyzed inflation and GDP separately, often using univariate models, despite calls for joint monitoring approaches (Mbeah-Baiden, 2013; SACU, 2013). While various statistical models such as regression, time series, Vector Auto-regressive (VAR), and Auto-regressive Integrated Moving Average (ARIMA) have been applied to Ghana's macroeconomic variables (Agalega & Antwi, 2013; Asenso et al., 2017), these models are primarily designed for forecasting or understanding dynamic relationships rather than for ongoing, real-time surveillance of macroeconomic health.

In contrast, methods drawn from Statistical Process Control (SPC), in particular multivariate control charts offer a complementary approach better suited for continuous monitoring and early anomaly detection. SPC originated in industrial quality control, but recent applications have demonstrated its utility beyond manufacturing. Multivariate control charts such as Hotelling's T2 chart, Multivariate CUSUM (MCUSUM), and Multivariate Exponentially Weighted Moving Average (MEWMA) are especially powerful when two or more correlated variables must be watched simultaneously (Quality Magazine, 2024).

The advantages of multivariate control charts over univariate monitoring approaches are well established in the quality-control and process-monitoring literature. First, multivariate charts enable the simultaneous monitoring of multiple interrelated indicators while incorporating the covariance structure between them, thereby providing a more holistic and statistically coherent view of system behaviour than separate univariate charts (Montgomery, 2019; Fuchs & Runger, 2010). Second, because they define a single joint control region, multivariate charts preserve the intended overall Type I error rate, whereas applying separate univariate charts to correlated variables can inflate false-alarm probabilities and distort control behaviour (Durfee, 1994; Phaladiganon et al., 2010). Third, cumulative-based schemes such as the Multivariate Cumulative Sum (MCUSUM) and the Multivariate Exponentially Weighted Moving Average (MEWMA) are particularly sensitive to small and persistent shifts in the process mean vector—subtle drifts that may remain undetected by Shewhart-type charts or by traditional econometric forecasting models designed primarily for prediction rather than continuous structural supervision (Lowry et al., 1992; Fallahnezhad & Ghalichehbaf, 2023).

Moreover, Statistical Process Control (SPC) techniques are widely recognized for supporting rapid or near real-time monitoring because they produce immediate visual signals that allow for prompt corrective action—an attribute increasingly relevant for macroeconomic surveillance and early-warning systems (Singh et al., 2002; SPC Software, 2023). Given the complexity and interdependence of macroeconomic variables such as GDP and inflation, multivariate control charts offer a proactive analytical tool for detecting emerging structural changes or early signs of instability rather than relying solely on periodic forecasts or ex post statistical evaluations (Yeganeh et al., 2023; Arciszewski, 2023).

This study addresses this gap by applying Hotelling's $T^2$, Multivariate Cumulative Sum (MCUSUM), and Multivariate Exponentially Weighted Moving Average (MEWMA) control charts to jointly monitor Ghana's GDP and inflation from 1973 to 2022. The objectives are to apply each method individually, compare their performance, and identify the most effective technique for macroeconomic monitoring. The paper seeks to achieve two specific objectives;

i. To jointly monitor Ghana's GDP and inflation rate using Hotelling's $T^2$, MCUSUM, and MEWMA control charts.

ii. To compare the performance of these methods and identify the most effective approach for monitoring GDP and inflation.

## REVIEW OF LITERATURE

### Empirical Studies

Recent empirical studies on Ghana's macroeconomic dynamics highlight persistent challenges related to inflation volatility, exchange rate depreciation, fiscal imbalances, and their combined effects on economic growth. These studies provide important context for jointly monitoring GDP and inflation using multivariate control charts.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

Ghana has experienced episodes of prolonged high inflation particularly during 2014–2016 and again after 2020 which was driven by exchange-rate depreciation, supply constraints, and fiscal pressures (Ackah & Opoku, 2023; Osei-Assibey & Adu, 2022). Several studies show that inflation in Ghana is often cost-push in nature, originating from imported inflation and energy price shocks rather than excess demand (Bawumia & Abradu-Otoo, 2021). This supports the need for real-time monitoring frameworks that can capture abrupt or structural shifts in inflation behaviour.

The relationship between inflation and economic growth has also been widely examined. Empirical findings generally suggest that moderate inflation may be compatible with growth, but high or unstable inflation significantly undermines economic performance in Ghana (Frimpong & Oteng-Abayie, 2010; Aboagye & Oteng-Abayie, 2020). For instance, Frimpong and Oteng-Abayie (2010) identified a threshold effect, where inflation above approximately 11% exerts a negative impact on growth. More recent analyses confirm that inflation's influence on GDP is asymmetric: inflation spikes have stronger negative effects on growth than disinflation episodes have positive effects (Ocran & Wiafe, 2021).

Studies focusing on macroeconomic stability indicators further show that GDP growth in Ghana is sensitive to combined shocks involving inflation, exchange rate movements, and fiscal deficits (Adom & Fiador, 2022; Boakye & Ackah, 2023). Importantly, these studies mostly rely on VAR, ARDL, or regression-based frameworks, which, while useful for forecasting and long-run relationships, are not designed for continuous process monitoring or early detection of abnormal behaviour. This methodological gap underscores the potential value of applying Statistical Process Control (SPC) techniques to Ghana's macroeconomic variables.

Integrating SPC methods, particularly multivariate control charts, offers a novel way to monitor the joint behaviour of GDP and inflation; variables that have demonstrated significant co-movement during periods of macroeconomic stress. Given the documented instability in Ghana's inflation dynamics and its measurable impact on economic activity, multivariate monitoring tools may help detect unusual shifts more rapidly than traditional econometric models, thereby supporting proactive policy responses.

**Theoretical Framework**

Multivariate control charts, extensions of univariate charts, are used to simultaneously monitor multiple related process variables, especially when they exhibit high cross-correlation (Mohmoud & Maravelakis, 2013; Montgomery, 2005). Widely applied in sectors like manufacturing and pharmaceuticals, these charts help detect small to moderate shifts in the process mean vector more effectively than separate univariate charts, particularly when variables are dependent (Alt, 1988; Montgomery, 1991). They operate by plotting each sample's test statistic against an upper control limit (UCL), with points above the UCL indicating potential process issues.

The MCUSUM chart, a multivariate extension of the univariate CUSUM, is designed to improve the sensitivity of the Hotelling's $T^2$ chart in detecting small to moderate shifts in the process mean vector by using accumulated data from previous observations (Crosier, 1988; Busaba et al., 2012a, 2012b). Various methods for constructing MCUSUM charts have been proposed, including approaches by Healy (1987), Woodall and Ncube (1985), Crosier (1988), and Pignatiello and Runger (1990). The MEWMA chart, proposed by Roberts et al. (1959) as the multivariate version of EWMA, monitors shifts using weighted averages of past observations and is highly sensitive to small and moderate process changes (Montgomery, 2005). Enhancements and applications have been studied in various contexts, such as clinical trials and VAR (1) processes (Khoo et al., 2006; Joner et al., 2008; Mahmoud & Zahran, 2010; Patel & Divecha, 2013), with modifications like the BMEWMA percentile approach addressing control limit selection issues (Fricker, 2007).

Multivariate Shewhart control charts, though widely used for detecting large process changes over 1.5σ (Mahmoud et al., 2015), are less effective for small to moderate changes and can produce more false alarms when normality assumptions are violated (Montgomery, 2009; Phaladiganon et al., 2011). In contrast, CUSUM and EWMA-type charts are more suited for detecting smaller shifts (Yeh et al., 2006). Average Run Length (ARL) measures the performance of these charts, with high in-control ARL minimizing false alarms and low out-of-control ARL enabling quicker detection of process changes (Montgomery, 2005; Pham, 2006).

### Related Works

Research comparing multivariate and univariate models shows that multivariate approaches often perform better for forecasting and risk management, especially with sufficiently large sample sizes (Siaw, 2014; Santos et al., 2009, 2013; Christoffersen, 2009). For example, multivariate GARCH models improve portfolio value-at-risk predictions, and bi-variate CUSUM methods detect small and moderate mean shifts faster than Shewhart charts (Woodall & Ncube, 1985). Variations of multivariate CUSUM by Crosier (1988), Healy (1987), and Pignatiello & Runger (1990) have comparable performance, though each has strengths in detecting specific types of shifts. In practice, MEWMA and MCUSUM charts have been effective for detecting modest changes even when quality indicators are uncorrelated (Fricker, 2007). Methodological advances include bootstrap-based control limits, generalized variance approaches, and boosting methods for identifying out-of-control variables (Phaladiganon et al., 2011; Yeh et al., 2006; Alfaro et al., 2009), as well as innovations in handling auto-correlated processes (Kalgonda, 2013, 2015; Gandy & Kvaløy, 2013).

Inflation is the sustained rise in prices, which erodes purchasing power and can signal poor economic management when rates are high, as it often leads to low growth and higher borrowing costs (McConnel & Brue, 2008; Asiedu & Lien, 2004). GDP, defined as the total market value of goods and services produced in a given period, is a central measure of economic performance, used for budgeting, forecasting, and assessing the overall health of an economy (Bade, 2016; Mankiw & Taylor, 2007). Both variables are key macroeconomic indicators, essential for understanding and managing a nation's economic trajectory.

This study applies Statistical Process Control (SPC) techniques, Hotelling's $T^2$, Multivariate Cumulative Sum (MCUSUM), and Multivariate Exponentially Weighted Moving Average (MEWMA) control charts to jointly monitor Ghana's Gross Domestic Product (GDP) and inflation rate, a method rarely used in macroeconomic analysis. In contrast to prior research that primarily relied on regression, univariate time series, or Vector Auto-regressive (VAR) models examining each variable separately, this approach employs multivariate SPC tools, widely used in industrial quality control, to capture the interdependence between GDP and inflation and detect both significant and subtle shifts in their joint mean vector. The comparative performance assessment shows Hotelling's $T^2$ as the most effective for early shift detection, with MCUSUM and MEWMA offering additional sensitivity to smaller changes, providing policymakers with a more robust framework for timely and accurate economic monitoring in Ghana.

## METHODS

### Data and Source

This study utilized annual secondary data covering the period 1973–2022. Data on Gross Domestic Product (GDP) and inflation rate were obtained from official publications of the Bank of Ghana (BOG). Specifically, GDP data were extracted from the National Accounts Statistical Series, and inflation data from the Consumer Price Index (CPI) Statistical Reports. The datasets are publicly accessible through the Bank of Ghana's Statistical Database (https://www.bog.gov.gh/) (Bank of Ghana, 2023). The data provide consistent, official macroeconomic indicators suitable for applying multivariate control charts over the 50-year period.

### Basic Time Series Model

The basic time series model is denoted as (AR (1)), being the first order Auto-regression model were considered. The purpose of the AR(1) model is to remove the effects of auto-correlation that may affect the chart. The de-trended data from the AR (1) model were used for the control chart. The value of $y$ at time $t$ is a linear function of the value of $y$ at time $t-1$. Since the data was collected over time, we consider a basic time series model (AR (1)) of the form;

$$y_t = \delta + \phi y_{t-1} + \varepsilon_t \qquad (3.1),$$

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

For GDP, we have

$$y_{1t} = \delta_1 + \phi_1 y_{1t} + \varepsilon_{1t} \quad (3.2),$$

For inflation, we have

$$y_{2t} = \delta_2 + \phi_2 y_{2t} + \varepsilon_{2t} \quad (3.3),$$

where $t$ = 1, 2, …, 50, $Y_{1t}$ is the GDP, $\delta_1$ is the intercept for GDP, $\Phi_1$ is the coefficient of the auto-regressive term, $\varepsilon_{1t}$ is the error (residuals) for GDP, $Y_{2t}$ is the inflation rate, $\delta_2$ is the intercept for inflation rate, $\Phi_2$ is the coefficient of the auto-regressive term and $\varepsilon_{2t}$ is the error (residuals) for inflation rate. The errors were then used for monitoring.

**Testing the Stationarity of Time Series**

In order to appropriately build a model, all series that are used in the analysis must be stationary. Since non-stationarity levels leads to spurious results, there is the need to make them stationary. The non-stationarity and the stationarity process can be distinguished by employing the Augmented Dickey – Fuller (ADF) unit-root test to check the Stationarity of the time series (Sim et al., 1990).

**Model Specification**

The models applied in this study were Hoteling $T^2$ , MCUSUM and MEWMA control charts. These joint models were used to monitor GDP and inflation rate growth of Ghana.

**The Estimation of Parameters**

The AR (1) model although, is fundamental to time series data, and is easy to estimate the parameters. Also, the number of lags when determine, the one with the minimum AIC value is chosen. The AIC value if not minimised with the same model, the likelihood ratio (LR) test method is applied (Johansen, 1995).

**Hoteling $T^2$ Control Chart**

The Hoteling $T^2$ control chart is the most well-known multivariate chart presently in use in the literature. (Montgomery, 2009) observed that the Hoteling $T^2$ chart was a trailblazer in the field of multivariate quality control research. Hotelling used multivariate control approach with data that included bomber location information during World War II.

The first assumption made by Hotelling was that the quality characteristics belonged to a normal multivariate distribution, with a covariance matrix S and a mean vector $\overline{X}$ . As a result, samples of size $n$ for each of the $p$ variables to be tracked and the estimates of the parameters are used to create the equation for calculating the estimates of the statistic $T^2$

$$T^2 = \left(X - \overline{X}\right)' S^{-1} \left(X - \overline{X}\right) \quad (3.4),$$

where $s^{-1}$ and $\overline{X}$ stand for the inverse of the covariance matrix and the estimates for the vector of means, respectively. The Hoteling $T^2$ chart for individual observations is built using expression (3.4) as a foundation

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

(Lowry et al., 1995). On the other hand, the statistics $T^2$ in phase (I) validate the control and are stated by following, the $F$ distribution with $p$ and $(mn- m- p+1)$ degrees of freedom.

The phase (I) monitoring of the process involve collecting information that is sufficient to determine whether or not the process reveals an in-control situation based on historical data. The upper control limit of the phase (I) monitoring scheme is expressed below;

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha, p, mn-m-p+1} \quad (3.5),$$

Also, based on the control limits calculated from Phase (I), Phase (II) which is the future observations are monitored to determine if the process continues to be stable or not. The upper control limit of the phase (II) monitoring scheme is expressed below;

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha, p, mn-m-p+1} \quad (3.6),$$

where the superior percentage point of α to a distribution is denoted by $F_{\alpha, p, mn-m-p+1}$. $F$, $p$, $m$, and $n$ denote the number of samples, sample size, and number of quality characteristic (variables). The process is considered to be out of statistical control if the value of $T^2$ is greater than the upper control limit (UCL). For both stages, the lower control limit (LCL) is zero.

However, it is worth mentioning that the use of the cumulative sum of difference in terms of their mean was examined by (Sullivan and Woodall, 1996), while the difference among consecutive observations instead of the difference respecting the mean was proposed by (Holmes and Mergen, 1993).

**MCUSUM Control Chart**

One of the test statistics for the MCUSUM techniques that (Crosier, 1988) gave, with the version that performs better in the *ARL* is;

$$p_k^2 = \left[ s_k' \left( \frac{\Sigma}{n} \right)^{-1} s_k \right]^{1/2} > h_1, \quad (3.7),$$

where $k = 1, 2, 3, \ldots$ an out-of-control alarm is signal if $p^2 > h_1$, where the control limit is $h_1$. The value of $h_1$ can be obtained by simulation (Alves et al., 2010) and (Fricker et al., 2008)). The proposed MCUSUM by (Crosier, 1988) is derived by replacing the scalar quantities of a uni-variate CUSUM by vectors. The control chart may be expressed as follows:

$$C_k = \left[ \left( S_{k-1} + X_{k} - \mu_0 \right)' \left( \frac{\Sigma}{n} \right)^{-1} \left( S_{k-1} + X_{k} - \mu_0 \right) \right]^{1/2} \quad (3.8),$$

$k = 1, 2, 3, \ldots$ where covariance matrix is Σ and $s_k$ are the cumulative sums as determined by:

$$S_k = \begin{cases} 0 & if\ c_k \le m \\ (S_{k-1} + X_k - \mu_0)\left(1 - \frac{m}{c_k}\right) & if\ c_k > m \end{cases} \quad (3.9),$$

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

where $m > 0$, is the reference value, the target value for the mean vector is $\mu_0$ and $s_k = 0$ (Khoo et al., 2009). The selection of m given by (Crosier, 1988) is $m = \theta/2$, where θ is defined in equation (3.14), and this appears to minimize the *ARL* at θ for a given in-control *ARL*.

**MEWMA Control Chart**

(Lowry et al., 1992) developed a MEWMA control chart. The MEWMA control chart employs weighted averages of previously observed random vectors to monitor the mean vector of the process. Consider that a *p*-dimensional random vector $\boldsymbol{X_k} = (\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, X_p)'$ has random variables as its components at time *k*, then the MEWMA control chart is defined as;

$$Z_k = \lambda X_k + (1-\lambda) X_{k-1} \qquad (3.10),$$

where $k = 1, 2, 3 \ldots$ and $Z_k$ is the weighted average of the time *k*. Consider that $Z_0 = 0$ where λ is the diagonal *p* x *p* matrix of the smoothing constant with $(0 < \lambda \leq 1)$ being the weighting constant and $Z_0$, which is the mean vector in-control of the process. Thus, the MEWMA control chart has a test statistic;

$$P^2 = Z'_k \Sigma_{Z_k}^{-1} Z_k > h_2 \qquad (3.11),$$

where a defined control limit is denoted by $h_2$. Every time $P^2$ surpasses a predetermined control limit, which is calibrated to produce a desired average time between false signals (ATFS), the MEWMA sounds an alarm. If $P^2$ does not exceed $h_2$ then the MEWMA move a new observation vector, and iterates through the next time step, recalculating the test statistic. The process continues until when $P^2 > h_2$. Since the test statistic is always non-negative as a result, only the upper control limit (*UCL*) is needed to monitor. The covariance of $(\sum z_k)$ is dependent upon the number of samples taken, the *UCL* will also depend on *k*. More so, the lower control limit (*LCL*) for the two phases is equal to zero in multivariate setting.

The two approaches provided by (Lowry et al., 1992), for estimating the $\Sigma_z$, is shown below; with the exact covariance matrix given as;

$$\Sigma_{Z_k} = \left\{ \frac{\lambda\left[1-(1-\lambda)^{2k}\right]}{(2-\lambda)} \right\} (\Sigma_x) \qquad (3.12),$$

Also, the asymptotic covariance matrix is given as;

$$\Sigma_{Z_k} = \left( \frac{\lambda}{2-\lambda} \right) (\Sigma_x) \qquad (3.13),$$

It is shown in literature that equation (3.9) performed better. Furthermore, (Lowry et al., 1992) again suggested that, the efficiency of the MEWMA chart in terms of the *ARL* relies on the mean vector $\mu$ and covariance matrix $\Sigma_z$, solely through the value of the non-centrality parameter θ, and

$$\theta = \left[ (\mu_1 - \mu_0)' \Sigma (\mu_1 - \mu_0) \right]^{1/2} \qquad (3.14),$$

Large values of $\theta$ corresponded, in general, to larger mean changes. The statistical process control situation that is under control is represented by $\theta = 0$. Keep in mind that, with the exception of very large values of θ or big shifts, *ARLs* typically tend to grow as λ increases for a given shift size. However, a special case of the MEWMA control chart is transformed on $P^2$ chart when $\lambda = 1$, it leads to $Z_K = X_K$ and $p^2 = X'_k \sum_x^{-1} X_k$. This is simply known as the $\chi^2$ chart or multivariate Shewhart control chart. Also, MEWMA with $\lambda < 1$ is more sensitive to minor alterations in the mean vector. From *Equation* (3.10) it is obvious that, when $Z_k$ is expanded recursively the expression below is obtained;

$$Z_k = \lambda X_k + \lambda(1-\lambda) X_{k-1} + \lambda(1-\lambda)^2 X_{k-2} + \ldots + \lambda(1-\lambda)^{k-1} X_1 + \lambda(1-\lambda)^k z_0 \quad (3.15),$$

Thus, $Z_k$ takes on a geometric form and is the weighted average of the time $k$ quality measurements that are provided with weights. The chart is known as multivariate exponentially weighted moving average in the literature.

**Limitations**

Although the dataset spans 50 years, the annual frequency may limit the sensitivity of multivariate control charts, particularly for detecting short-term fluctuations or seasonal effects. Control chart parameters estimated from a single annual time series may not fully capture intra-year variability (Montgomery, 2019).

Using only BOG-reported data may introduce source-specific biases or reporting inconsistencies. Cross-validation with other databases (e.g., Ghana Statistical Service) was not performed, which may affect robustness.

Ghana experienced major economic events, including high inflation periods, currency devaluations, and fiscal adjustments. These structural breaks may affect the assumption of process stability and the estimation of control limits (Pham, 2006; Montgomery, 2019).

To mitigate potential bias, control limits for Hotelling's $T^2$, MCUSUM, and MEWMA charts were estimated using standard procedures from multivariate SPC literature, with in-control process assumptions checked via historical baseline periods (Lowry et al., 1992; Fallahnezhad & Ghalichehbaf, 2023).

By acknowledging these limitations, the study ensures transparency and guides interpretation of the findings, emphasizing that multivariate control charts are applied as complementary monitoring tools rather than definitive predictive models.

## RESULTS AND DISCUSSIONS

**Descriptive Statistics**

The descriptive statistics is shown below in Table 4.1. The statistics include mean, standard deviation, skewness, and excess Kurtosis of Gross Domestic Product (GDP) and inflation rate. The Gross Domestic Product (GDP) and inflation rate have positive average values (means) of about 20.67 and 28.99, respectively, with standard deviations of 24.09 and 26.45, respectively.

The results from the descriptive statistics also showed that the Gross Domestic Product (GDP) is positively skewed (skewness value of 1.238), which further demonstrates that the majority of Gross Domestic Product (GDP) values are less than their means. The excess kurtosis for Gross Domestic Product (GDP) is negative making it platykurtic (-0.008). Hence, its peak is flatter than the typical normal distribution. Table 4.1 also showed that Ghana's inflation rate is positively skewed (skewness value of 2.200), indicating that the majority of inflation rate values are less than their means. The excess kurtosis for inflation rate is positive making it leptokurtic (4.814), hence exhibiting more peaked values than the normal distribution. Table 4.1 presents summary of the results.

Table 4.1: Summary of Descriptive Statistics of the Variables for Ghana

| **Variable** | **N** | **Min** | **Max** | **Mean** | **Std. Dev.** | **Skewness** | **Excess kurtosis** |
|---|---|---|---|---|---|---|---|
| GDP | 50 | 2.7700 | 79.1600 | 20.6666 | 24.0885 | 1.2380 | -0.0080 |
| Inflation | 50 | 8.6800 | 121.3500 | 26.4480 | 26.4488 | 2.2000 | 4.8140 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

**Testing the Stationarity of Time Series**

Table 4.2 showed the results for the stationarity test of the original data on both GDP and inflation for the ADF test. The output shows non-stationarity on GDP since the p-values is greater than 0.05 as shown below.

Table 4.2: Stationarity Test of the Original Variables

| **Gross Domestic Product** | | | **Inflation Rate** | |
|---|---|---|---|---|
| **ADF Test** | **Statistic** | **P-value.** | **Statistic** | **P-value** |
| | -0.5876 | 0.9741 | -3.5670 | 0.0448 |

Table 4.3 showed the results for the stationarity test of the residual on both GDP and inflation for the ADF test. The output shows stationarity on both GDP and inflation rate since the p-values are all less than 0.05 as shown below.

Table 4.3: Stationarity Test of the Residual Variables

| **GROSS DOMESTIC PRODUCT** | | | **INFLATION RATE** | |
|---|---|---|---|---|
| **ADF Test** | **Statistic** | **P-value.** | **Statistic** | **P-value** |
| | -4.3578 | 0.0100 | -3.2722 | 0.0400 |

**Results for AR (1) Model**

Table 4.4 showed the parameters estimate of the AR (1) model for GDP. The AR (1) for GDP had an intercept of 2.6463 and a slope of 0.9936 which indicates stationarity (since $\boldsymbol{\phi_1} < 1$).

Table 4.4: AR (1) Model Parameter Estimate for GDP

| **Parameter** | **Coefficient** | **se** | **Log-likelihood** | **AIC** |
|---|---|---|---|---|
| Intercept($\delta_1$) | 2.6463 | 1.3869 | | |
| $\phi_1$ | 0.9936 | 0.0085 | 15.71 | -25.42 |

Table 4.5 showed the parameters estimate of the AR (1) model for inflation rate. The AR(1) for inflation rate had an intercept of 3.0850 and a slope value of 0.5940 which indicates a stationarity (since $\phi_2 < 1$)

Table 4.5: AR (1) Model Parameter Estimate for Inflation Rate

| **Parameter** | **coefficient** | **se** | **Log-likelihood** | **AIC** |
|---|---|---|---|---|
| Intercept($\delta_2$) | 3.0850 | 0.1898 | | |
| $\phi_2$ | 0.5940 | 0.1111 | -42.25 | 90.49 |

Table 4.6 showed the output for Ljung Box test Statistic for the original data on both GDP and inflation. The output indicates that, there exist serial correlation on both GDP and inflation rate since the corresponding p-values are all less than 0.05 as shown below.

| **Variable** | **Chi-Square** | **P-value** |
|---|---|---|
| GDP | 220.7500 | 0.0000 |
| Inflation | 42.4400 | 0.0100 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

Table 4.6: Test of Serial Correlation of the Original Variables

| Variable | Chi-Square | P-value |
|---|---|---|
| GDP | 220.75 | 0.0000 |
| Inflation | 42.44 | 0.0100 |

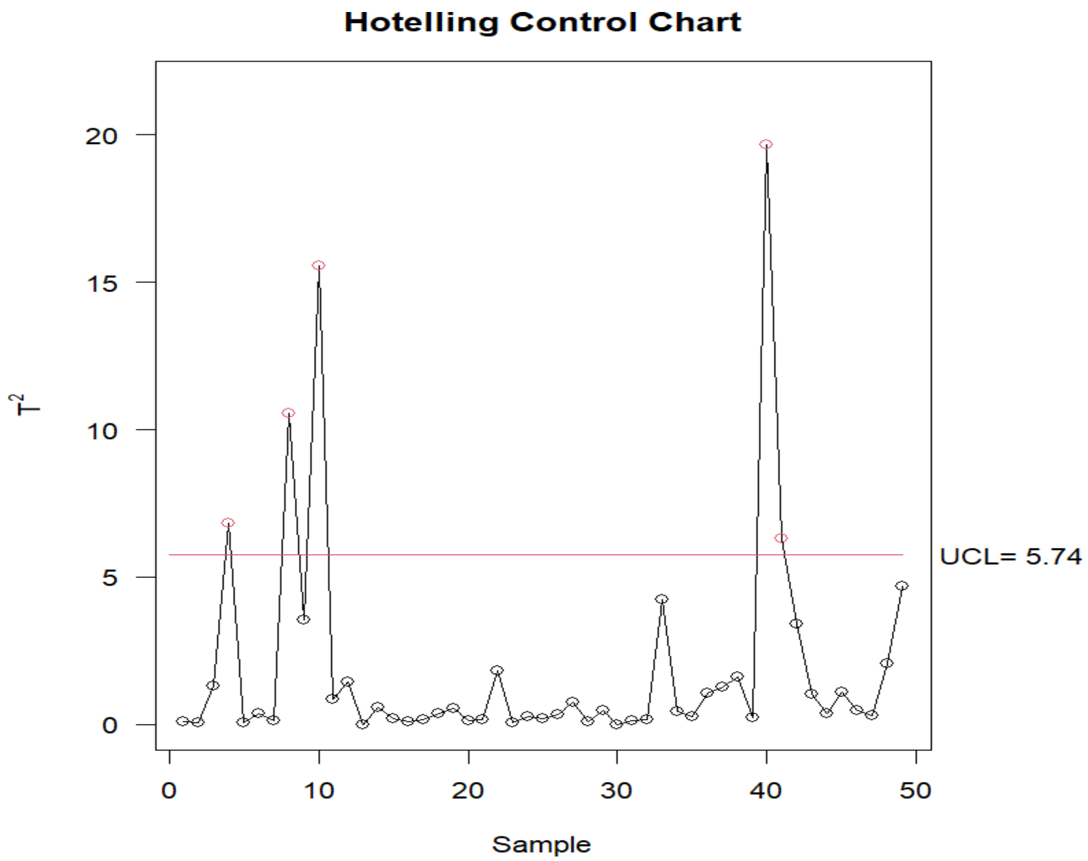
Table 4.7 also showed that, there is no serial correlation on both GDP and inflation rate owing to the fact that, the corresponding p-values are greater than 0.05 as shown below.

Table 4.7: Serial Correlation of the Residuals

| Variable | Chi-Square | p-value |
|---|---|---|
| GDP | 16.7460 | 0.0802 |
| Inflation | 12.2400 | 0.2661 |

**Hoteling $T^2$ Control Chart Based on Sullivan and Woodall (SW) Method**

Figure 4.1 showed the output of the Hoteling $T^2$ control chart based on Sullivan and Woodall (SW) method. The control chart based on the macroeconomic variables such as Gross Domestic Product (GDP) and inflation rate given as, variable $X_1$ (GDP) and variable $X_2$ (inflation). It can be observed that, five samples (4, 8, 10, 40, and 41) which corresponding to the years 1976, 1980, 1982, 2012, and 2013 respectively, fall outside the control limit of 5.74 (Threshold), similar results can be seen in Table 4.6.




.

Figure 4.1: Hoteling $T^2$ control chart by SW for GDP and inflation rate.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

Table 4.8: Sullivan Woodall method for GDP and inflation rate

| Sample | error_GDP | error_inflation | $T^2$Chart(SW) |
|---|---|---|---|
| 1 | -1.05727 | -4.39226 | 0.11 |
| 2 | -1.01375 | 2.446733 | 0.06 |
| 3 | -0.97139 | 27.09586 | 1.29 |
| **4** | **-0.51022** | **62.29025** | **6.84** |
| 5 | -0.47257 | 4.782753 | 0.05 |
| 6 | -0.59638 | 14.40802 | 0.37 |
| 7 | -0.53697 | 8.626386 | 0.14 |
| **8** | **-1.20961** | **77.57967** | **10.55** |
| 9 | -1.15285 | -43.4044 | 3.56 |
| **10** | **-0.94756** | **94.00721** | **15.55** |
| 11 | -0.61815 | -21.1859 | 0.85 |
| 12 | -0.88844 | -27.2678 | 1.43 |
| 13 | 0.258918 | 1.197595 | 0.01 |
| 14 | -1.68754 | -14.4143 | 0.58 |
| 15 | -0.86784 | 9.664322 | 0.18 |
| 16 | -0.95166 | -5.14798 | 0.11 |
| 17 | -0.36313 | 9.537973 | 0.16 |
| 18 | -0.31195 | -14.4463 | 0.39 |
| 19 | -1.22283 | -15.324 | 0.55 |
| 20 | -1.48753 | 4.259274 | 0.13 |
| 21 | -1.5443 | -4.43241 | 0.18 |
| 22 | 0.000986 | 31.8673 | 1.81 |
| 23 | -0.56871 | 6.319687 | 0.08 |
| 24 | -1.09253 | -9.88055 | 0.26 |
| 25 | -0.46135 | -9.60859 | 0.19 |
| 26 | -0.8287 | -12.5049 | 0.34 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | |
|---|---|---|---|
| 27 | -3.81576 | 1.169894 | 0.76 |
| 28 | -0.66519 | 6.841288 | 0.09 |
| 29 | -0.1449 | -16.3965 | 0.49 |
| 30 | 0.429817 | 0.399357 | 0.01 |
| 31 | 0.176891 | -9.00633 | 0.14 |
| 32 | 0.750139 | -8.98881 | 0.16 |
| 33 | 8.985451 | -12.6163 | 4.24 |
| 34 | 2.477024 | -10.5136 | 0.45 |
| 35 | 2.271181 | -4.95681 | 0.29 |
| 36 | -4.32202 | -4.44792 | 1.07 |
| 37 | 4.535311 | -14.2105 | 1.27 |
| 38 | 5.34449 | -12.5376 | 1.61 |
| 39 | -0.07544 | -11.2455 | 0.23 |
| **40** | **19.48782** | **-9.03712** | **19.68** |
| **41** | **-10.7358** | **-6.17445** | **6.31** |
| 42 | -7.8294 | -6.30507 | 3.43 |
| 43 | 4.448486 | -6.50476 | 1.04 |
| 44 | 1.750025 | -11.8648 | 0.36 |
| 45 | 4.265067 | -12.9629 | 1.11 |
| 46 | -1.78751 | -11.8243 | 0.47 |
| 47 | -1.15809 | -10.4247 | 0.3 |
| 48 | 6.211929 | -11.0048 | 2.07 |
| 49 | -9.49622 | 10.53844 | 4.68 |

* The bolden shows out-of-control points

## Hoteling $T^2$ Control Chart Based on Holm and Mergen (HM) Method

Figure 4.2 showed the output of the Hotelling $T^2$ control chart based on Holm and Mergen method. The control chart based on the macroeconomic variables such as Gross Domestic Product (GDP) and inflation rate were monitored given as, variable $X_1$ (GDP) and variable $X_2$ (inflation). It can be observed that, four samples (4, 8, 10, and 40) which correspond to the years 1976, 1980, 1982, and 2012, respectively, fall outside the control limit of 5.74 (Threshold), similar results can be seen in Table 4.7.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

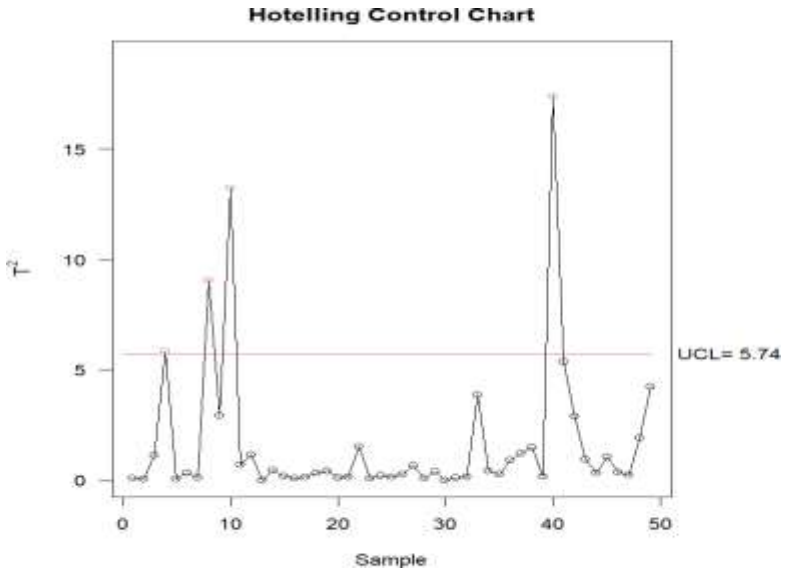



Figure 4.2: Hoteling $T^2$ control chart by HM for the GDP and inflation rate.

Table 4.9: Holm Mergen method for GDP and inflation rate

| Sample | error_GDP | error_inflation | $T^2$Chart(HM) |
|---|---|---|---|
| 1 | -1.05727 | -4.39226 | 0.08 |
| 2 | -1.01375 | 2.446733 | 0.05 |
| 3 | -0.97139 | 27.09586 | 1.13 |
| **4** | **-0.51022** | **62.29025** | **5.82** |
| 5 | -0.47257 | 4.782753 | 0.04 |
| 6 | -0.59638 | 14.40802 | 0.32 |
| 7 | -0.53697 | 8.626386 | 0.12 |
| **8** | **-1.20961** | **77.57967** | **9.04** |
| 9 | -1.15285 | -43.4044 | 2.92 |
| **10** | **-0.94756** | **94.00721** | **13.23** |
| 11 | -0.61815 | -21.1859 | 0.70 |
| 12 | -0.88844 | -27.2678 | 1.17 |
| 13 | 0.258918 | 1.197595 | 0.01 |
| 14 | -1.68754 | -14.4143 | 0.46 |
| 15 | -0.86784 | 9.664322 | 0.17 |
| 16 | -0.95166 | -5.14798 | 0.08 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | |
|---|---|---|---|
| 17 | -0.36313 | 9.537973 | 0.14 |
| 18 | -0.31195 | -14.4463 | 0.32 |
| 19 | -1.22283 | -15.324 | 0.43 |
| 20 | -1.48753 | 4.259274 | 0.12 |
| 21 | -1.5443 | -4.43241 | 0.14 |
| 22 | 0.000986 | 31.8673 | 1.52 |
| 23 | -0.56871 | 6.319687 | 0.07 |
| 24 | -1.09253 | -9.88055 | 0.21 |
| 25 | -0.46135 | -9.60859 | 0.15 |
| 26 | -0.8287 | -12.5049 | 0.27 |
| 27 | -3.81576 | 1.169894 | 0.67 |
| 28 | -0.66519 | 6.841288 | 0.09 |
| 29 | -0.1449 | -16.3965 | 0.41 |
| 30 | 0.429817 | 0.399357 | 0.01 |
| 31 | 0.176891 | -9.00633 | 0.12 |
| 32 | 0.750139 | -8.98881 | 0.14 |
| 33 | 8.985451 | -12.6163 | 3.86 |
| 34 | 2.477024 | -10.5136 | 0.43 |
| 35 | 2.271181 | -4.95681 | 0.27 |
| 36 | -4.32202 | -4.44792 | 0.90 |
| 37 | 4.535311 | -14.2105 | 1.21 |
| 38 | 5.34449 | -12.5376 | 1.50 |
| 39 | -0.07544 | -11.2455 | 0.19 |
| **40** | **19.48782** | **-9.03712** | **17.40** |
| 41 | -10.7358 | -6.17445 | 5.37 |
| 42 | -7.8294 | -6.30507 | 2.90 |
| 43 | 4.448486 | -6.50476 | 0.95 |
| 44 | 1.750025 | -11.8648 | 0.34 |

INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | |
|---|---|---|---|
| 45 | 4.265067 | -12.9629 | 1.05 |
| 46 | -1.78751 | -11.8243 | 0.37 |
| 47 | -1.15809 | -10.4247 | 0.23 |
| 48 | 6.211929 | -11.0048 | 1.91 |
| 49 | -9.49622 | 10.53844 | 4.23 |

* The bolden shows out-of-control points

**Decomposition of the $T^2$ Chart**

The decomposition issue was utilized to identify which quality features (variables) were in charge for the variation when an out-of-control signal occurred. Nonetheless, (Mason et al., 1995) approach is the one that is most commonly accepted to address the decomposition problem. Table 4.8 and 4.9 showed results of decomposition analysis for Sullivan and Woodall (SW) method and Holm and Mergen (HM) method. From Table 4.8 and 4.9, it revealed that the combined influence variable is represented by the third decomposed row, with the first two decomposed rows representing each of the variables investigated. The variable $X_2$ (inflation) indicates the source of variability for the out-of-control situation at time point 4, which correspond to the year 1976. It follows that, variable $X_2$ (inflation) went out-of-control situation since it is significant ($p$-value $< 0.05$) for the period 1976.

The variable $X_2$ (inflation) indicate the source of variability for the out-of-control situation at time point 8, which correspond to the years 1980. This means that, variable $X_2$ (inflation) went out-of-control situation since it is significant ($p$-value $< 0.05$) for the period 1980. Again, the variable $X_2$ (inflation) indicate the source of variability for the out-of-control situation at time point 10, which correspond to the years 1982. This means that, variable $X_2$ (inflation) went out-of-control situation since it is significant ($p$-value $< 0.05$) for the period 1982.

In addition, the variable $X_1$ (GDP) reveals the source of variability for the out-of-control situation at time point 40, which correspond to the years 2012. It follows that, variable $X_1$ (GDP) went out-of-control situation since it is significant ($p$-value $< 0.05$) for the period 2012. The variable $X_1$ (GDP) again, reveals the source of variability for the out-of-control scenario at time point 41, which correspond to the years 2013. This also reveals that, variable $X_1$ (GDP) went out-of-control situation since it is significant ($p$-value $< 0.05$) for the period 2013.

Table 4.10: Decomposition of GDP and inflation rate by SW method

| Sample | $t^2$ | Decomp | UCL | P-value | GDP (X1) | Inflation (X2) |
|---|---|---|---|---|---|---|
| | [1,] | 0.0135 | 6.6988 | 0.9081 | 1 | 0 |
| 4 | [2,] | 6.7848 | 5.6988 | 0.0122 | 2 | 0 |
| | [3,] | 6.8413 | 8.7161 | 0.0024 | 1 | 2 |
| | [1,] | 0.0756 | 5.6988 | 0.7845 | 1 | 0 |
| 8 | [2,] | 10.5244 | 5.6988 | 0.0021 | 2 | 0 |
| | [3,] | 10.5513 | 8.7161 | 0.0002 | 1 | 2 |
| | [1,] | 0.0464 | 5.6988 | 0.8304 | 1 | 0 |

INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | | | | |
|---|---|---|---|---|---|---|
| 10 | [2,] | 15.4533 | 5.6988 | 0.0003 | 2 | 0 |
| | [3,] | 15.5543 | 8.7161 | 0.0000 | 1 | 2 |
| | [1,] | 19.6283 | 5.6988 | 0.0001 | 1 | 0 |
| 40 | [2,] | 0.1428 | 5.6988 | 0.7072 | 2 | 0 |
| | [3,] | 19.6775 | 8.7161 | 0.0000 | 1 | 2 |
| | [1,] | 5.9570 | 5.6988 | 0.0184 | 1 | 0 |
| 41 | [2,] | 0.0667 | 5.6988 | 0.7974 | 2 | 0 |
| | [3,] | 6/3084 | 8.7161 | 0.0037 | 1 | 2 |

Table 4.11: Decomposition of GDP and inflation rate by HM method

| Sample | $t^2$ | Decomp | UCL | P-value | GDP (X1) | Inflation (x2) |
|---|---|---|---|---|---|---|
| | [1,] | 0.0119 | 6.6988 | 0.9136 | 1 | 0 |
| 4 | [2,] | 5.8164 | 5.6988 | 0.0198 | 2 | 0 |
| | [3,] | 5.8169 | 8.7161 | 0.0055 | 1 | 2 |
| | [1,] | 0.0669 | 5.6988 | 0.7971 | 1 | 0 |
| 8 | [2,] | 9.0222 | 5.6988 | 0.0042 | 2 | 0 |
| | [3,] | 9.0442 | 8.7161 | 0.0005 | 1 | 2 |
| | [1,] | 0.0410 | 5.6988 | 0.8403 | 1 | 0 |
| 10 | [2,] | 13.2477 | 5.6988 | 0.0007 | 2 | 0 |
| | [3,] | 13.2524 | 8.7161 | 0.0000 | 1 | 2 |
| | [1,] | 17.3564 | 5.6988 | 0.0001 | 1 | 0 |
| 40 | [2,] | 0.1224 | 5.6988 | 0.7279 | 2 | 0 |
| | [3,] | 17.3953 | 8.7161 | 0.0000 | 1 | 2 |

**MCUSUM Control Chart**

Figure 4.3 showed the MCUSUM control chart based on macroeconomic variables such as Gross Domestic Product (GDP) and inflation rate were monitored. The chart was constructed using m = 0.5 and c = 5.5. It can be observed that, only one sample point (10) which correspond to the year 1982, falls outside the control limit of 5.5 (Threshold).

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

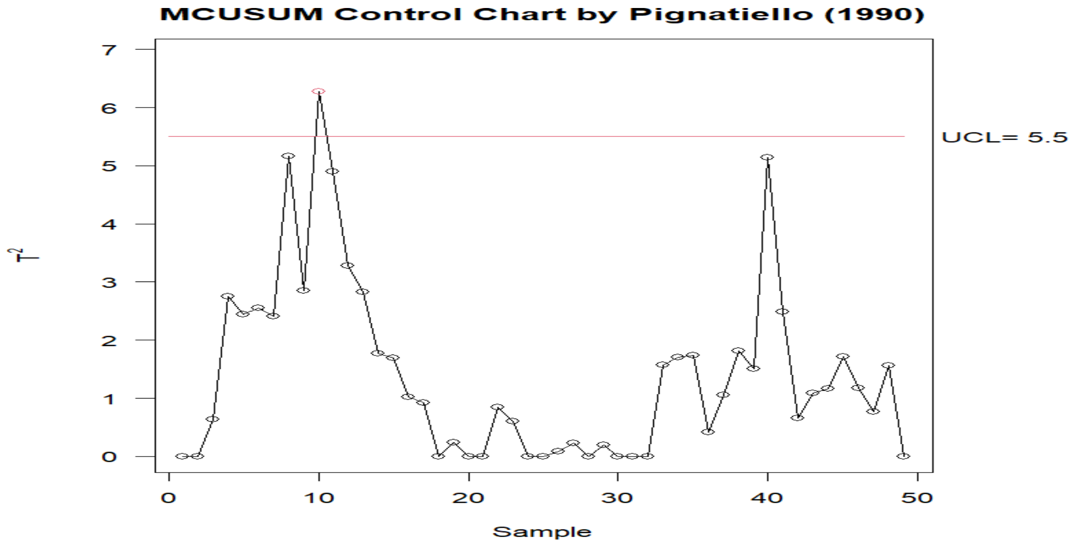



Figure 4.3: MCUSUM Control Chart for GDP and Inflation Rate

Table 4.12: MCUSUM Chart for GDP and inflation rate

| Sample | error_GDP | error_inflation | MCUSUM |
|---|---|---|---|
| 1 | -1.05727 | -4.39226 | 0.00 |
| 2 | -1.01375 | 2.446733 | 0.00 |
| 3 | -0.97139 | 27.09586 | 0.64 |
| 4 | -0.51022 | 62.29025 | 2.75 |
| 5 | -0.47257 | 4.782753 | 2.45 |
| 6 | -0.59638 | 14.40802 | 2.55 |
| 7 | -0.53697 | 8.626386 | 2.41 |
| 8 | -1.20961 | 77.57967 | 5.16 |
| 9 | -1.15285 | -43.4044 | 2.87 |
| **10** | **-0.94756** | **94.00721** | **6.28** |
| 11 | -0.61815 | -21.1859 | 4.91 |
| 12 | -0.88844 | -27.2678 | 3.31 |
| 13 | 0.258918 | 1.197595 | 2.85 |
| 14 | -1.68754 | -14.4143 | 1.88 |
| 15 | -0.86784 | 9.664322 | 1.81 |
| 16 | -0.95166 | -5.14798 | 1.23 |
| 17 | 9-0.36313 | 9.537973 | 1.10 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | |
|---|---|---|---|
| 18 | -0.31195 | -14.4463 | 0.23 |
| 19 | -1.22283 | -15.324 | 0.27 |
| 20 | -1.48753 | 4.259274 | 0.02 |
| 21 | -1.5443 | -4.43241 | 0.00 |
| 22 | 0.000986 | 31.8673 | 0.85 |
| 23 | -0.56871 | 6.319687 | 0.60 |
| 24 | -1.09253 | -9.88055 | 0.00 |
| 25 | -0.46135 | -9.60859 | 0.00 |
| 26 | -0.8287 | -12.5049 | 0.09 |
| 27 | -3.81576 | 1.169894 | 0.42 |
| 28 | -0.66519 | 6.841288 | 0.10 |
| 29 | -0.1449 | -16.3965 | 0.18 |
| 30 | 0.429817 | 0.399357 | 0.00 |
| 31 | 0.176891 | -9.00633 | 0.00 |
| 32 | 0.750139 | -8.98881 | 0.00 |
| 33 | 8.985451 | -12.6163 | 1.58 |
| 34 | 2.477024 | -10.5136 | 1.71 |
| 35 | 2.271181 | -4.95681 | 1.74 |
| 36 | -4.32202 | -4.44792 | 0.54 |
| 37 | 4.535311 | -14.2105 | 1.14 |
| 38 | 5.34449 | -12.5376 | 1.90 |
| 39 | -0.07544 | -11.2455 | 1.62 |
| 40 | 19.48782 | -9.03712 | 5.33 |
| 41 | -10.7358 | -6.17445 | 2.59 |
| 42 | -7.8294 | -6.30507 | 1.16 |
| 43 | 4.448486 | -6.50476 | 1.38 |
| 44 | 1.750025 | -11.8648 | 1.48 |
| 45 | 4.265067 | -12.9629 | 2.00 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | |
|---|---|---|---|
| 46 | -1.78751 | -11.8243 | 1.60 |
| 47 | -1.15809 | -10.4247 | 1.39 |
| 48 | 6.211929 | -11.0048 | 1.98 |
| 49 | -9.49622 | 10.53844 | 0.78 |

* The bolden shows out-of-control points

**MEWMA Control Chart**

Figure 4.4 also showed the MEWMA control chart based on macroeconomic variables such as Gross Domestic Product (GDP) and inflation rate were monitored. The chart was constructed such that p = 2, λ = 1, $ARL_0$ = 200. It can be observed that three samples points (8, 10, and 41) which correspond to the years 1980, 1982, and 2013 respectively, fall outside the control limit of 8.63 (Threshold).

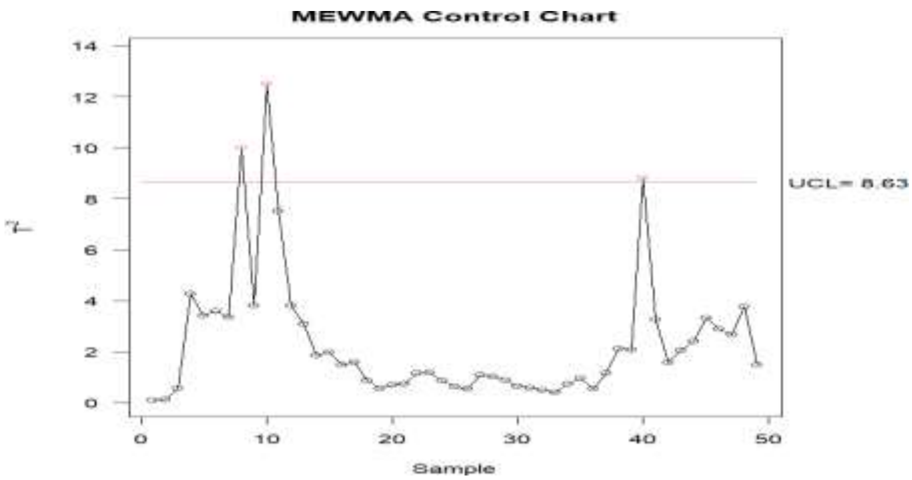



Figure 4.4: MEWMA control chart with λ = 1 using the GDP and inflation rate

Table 4.13: MEWMA Chart for GDP and inflation rate

| Sample | error_GDP | error_inflation | MEWMA |
|---|---|---|---|
| 1 | -1.05727 | -4.39226 | 0.11 |
| 2 | -1.01375 | 2.446733 | 0.12 |
| 3 | -0.97139 | 27.09586 | 0.56 |
| 4 | -0.51022 | 62.29025 | 4.27 |
| 5 | -0.47257 | 4.782753 | 3.42 |
| 6 | -0.59638 | 14.40802 | 3.60 |
| 7 | -0.53697 | 8.626386 | 3.35 |
| **8** | **-1.20961** | **77.57967** | **10.00** |
| 9 | -1.15285 | -43.4044 | 3.81 |

| | | | |
|---|---|---|---|
| **10** | **-0.94756** | **94.00721** | **12.51** |
| 11 | -0.61815 | -21.1859 | 7.52 |
| 12 | -0.88844 | -27.2678 | 3.81 |
| 13 | 0.258918 | 1.197595 | 3.09 |
| 14 | -1.68754 | -14.4143 | 1.86 |
| 15 | -0.86784 | 9.664322 | 2.00 |
| 16 | -0.95166 | -5.14798 | 1.49 |
| 17 | -0.36313 | 9.537973 | 1.59 |
| 18 | -0.31195 | -14.4463 | 0.87 |
| 19 | -1.22283 | -15.324 | 0.56 |
| 20 | -1.48753 | 4.259274 | 0.70 |
| 21 | -1.5443 | -4.43241 | 0.74 |
| 22 | 0.000986 | 31.8673 | 1.17 |
| 23 | -0.56871 | 6.319687 | 1.19 |
| 24 | -1.09253 | -9.88055 | 0.87 |
| 25 | -0.46135 | -9.60859 | 0.63 |
| 26 | -0.8287 | -12.5049 | 0.55 |
| 27 | -3.81576 | 1.169894 | 1.10 |
| 28 | -0.66519 | 6.841288 | 1.03 |
| 29 | -0.1449 | -16.3965 | 0.89 |
| 30 | 0.429817 | 0.399357 | 0.65 |
| 31 | 0.176891 | -9.00633 | 0.58 |
| 32 | 0.750139 | -8.98881 | 0.49 |
| 33 | 8.985451 | -12.6163 | 0.42 |
| 34 | 2.477024 | -10.5136 | 0.75 |
| 35 | 2.271181 | -4.95681 | 0.97 |
| 36 | -4.32202 | -4.44792 | 0.55 |
| 37 | 4.535311 | -14.2105 | 1.15 |

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

| | | | |
|---|---|---|---|
| 38 | 5.34449 | -12.5376 | 2.13 |
| 39 | -0.07544 | -11.2455 | 2.09 |
| **40** | **19.48782** | **-9.03712** | **8.82** |
| 41 | -10.7358 | -6.17445 | 3.26 |
| 42 | -7.8294 | -6.30507 | 1.59 |
| 43 | 4.448486 | -6.50476 | 2.06 |
| 44 | 1.750025 | -11.8648 | 2.41 |
| 45 | 4.265067 | -12.9629 | 3.33 |
| 46 | -1.78751 | -11.8243 | 2.89 |
| 47 | -1.15809 | -10.4247 | 2.68 |
| 48 | 6.211929 | -11.0048 | 3.78 |
| 49 | -9.49622 | 10.53844 | 1.49 |

*The bolden shows out-of-control points

**Performance Comparison of Hoteling $T^2$, MCUSUM and MEWMA Charts**

The performance of the charts **(**Hoteling $T^2$, MCUSUM and MEWMA chart**)** were compared. It can be observed From Figure 4.1, 4.2 and Table 4.8, 4.9 show that, the Hoteling $T^2$ control chart for Sullivan and Woodall (SW) signaled a shift in the mean vector at the time points (4, 8, 10, 40, and 41) which represents the years 1976, 1980, 1982, 2012, and 2013 respectively, whiles the Hoteling $T^2$ control chart for Holm and Mergen (HM) signaled a shift in the mean vector at the time points (4, 8, 10, and 40) which also represents the years 1976, 1980, 1982, and 2012 respectively. Both Sullivan and Woodall (SW), Holm and Mergen control charts have the same performance since the out-of-control signal (alarm) is detected earlier at the 4$^{th}$ sample at the control limit of 5.74.

On the other hand, MCUSUM control chart in Figure 4.3 and Table 4.10 detected the signal at the time point 10, representing the year 1982. These sample point experienced the signal (alarm) earlier at the 10$^{th}$ sample at the control limit of 5.5. MEWMA chart also, signaled earlier at the 8$^{th}$ sample at the control limit of 5.5, as seen in Figure 4.4 and Table 4.11. The control charts perform better when the signal (alarm) is spotted earlier. Clearly, the Hotelling $T^2$ control chart outperform both MCUSUM and MEWMA control chart. Meanwhile, MEWMA chart also outperform MCUSUM chart.

Furthermore, there may be a large shift in the Gross Domestic Product (GDP) and inflation rate data since Hotelling $T^2$ chart was faster in determining the mean shift, MCUSUM and MEWMA charts were able to identify small and moderate shifts in the mean vector.

## DISCUSSIONS

This study applied Hotelling's $T^2$, MCUSUM, and MEWMA control charts to jointly monitor Ghana's GDP and inflation rate from 1973 to 2022, evaluating their ability to detect unusual shifts in the macroeconomic environment. The charts revealed distinct out-of-control signals, which can be interpreted in the context of Ghana's historical economic events.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue XI November 2025

The Hotelling T² chart indicated out-of-control situations at sample points 4, 8, 10, 40, and 41, corresponding to the years 1976, 1980, 1982, 2012, and 2013, respectively. The MCUSUM chart ($k = 0.5$, $c = 5.5$) flagged only the 10th sample point (1982) as exceeding the control limit, whereas the MEWMA chart ($p = 2$, $\lambda = 1$, $ARL_0 = 200$) identified three points; 1980, 1982, and 2013 as out-of-control, exceeding the limit of 8.63.

**Interpretation of Out-of-Control Signals**

**1976 (Inflation, Hotelling T²):** Ghana faced high inflation, currency depreciation, and balance-of-payments pressures during this period (Aryeetey & Fosu, 2005). These macroeconomic challenges likely contributed to the observed out-of-control signal in inflation, reflecting instability in the joint GDP-inflation dynamics.

**1980 (Inflation, Hotelling T² and MEWMA):** The 1980 out-of-control signal coincides with severe internal and external debt pressures, exchange rate depreciation, and declining commodity prices (Antwi et al., 2014). These factors created heightened inflationary pressures, consistent with the MEWMA and Hotelling T² detection of abnormal joint behaviour.

**1982 (Inflation, Hotelling T², MCUSUM, MEWMA**): This period corresponds to ongoing economic difficulties, including increased government borrowing and fiscal imbalances, which exacerbated inflationary pressures (Frimpong & Oteng-Abayie, 2010). The convergence of all three charts in signaling this year underscores the persistence of small but sustained shifts in macroeconomic variables.

**2012 (GDP, Hotelling T²):** The out-of-control signal in GDP aligns with the pre-election fiscal expansion for the presidential and legislative elections. Although the economy had benefited from the discovery of offshore oil reserves in 2007, government spending surged in anticipation of elections, temporarily accelerating GDP growth beyond expected trends (Agyire-Tettey, 2017; Weiseke, 2019).

**2013 (GDP, Hotelling T² and MEWMA):** The subsequent fiscal year experienced post-election fiscal pressures, including increased domestic and external borrowing to finance election-related expenditures and debt servicing (Mbaye, 2019). This likely explains the observed out-of-control points for GDP, capturing the delayed effects of expansionary policies on macroeconomic stability.

**Comparative Performance of Control Charts**

Consistent with Moraes et al. (2015), the Hotelling T² chart performed best in detecting moderate to large shifts in the mean vector, capturing most major deviations in GDP and inflation. The MCUSUM chart, in contrast, was more sensitive to small, persistent shifts, detecting the 1982 inflation deviation even when Hotelling T² and MEWMA showed less responsiveness. The MEWMA chart effectively highlighted subtle drifts over time, such as the 1980 inflation and 2013 GDP shifts, corroborating prior findings on its utility for early detection of incremental changes (Custódio et al., 2013). Overall, the combined application of all three charts provides complementary insights: Hotelling T² for broad shifts, MCUSUM for accumulated small deviations, and MEWMA for time-weighted detection.

These findings demonstrate that multivariate SPC tools can provide a proactive monitoring framework for macroeconomic variables, enabling policymakers to detect emerging instabilities associated with major events such as fiscal expansions, debt accumulation, or election-related spending, well before traditional forecasts signal potential risk.

**Study Limitations and MCUSUM Performance**

While the multivariate control charts provided valuable insights into the joint behaviour of GDP and inflation in Ghana, several limitations should be acknowledged. First, the analysis relied on annual data, which may reduce the sensitivity of the control charts to short-term fluctuations or seasonal variations. Second, structural breaks, such as oil discoveries, fiscal expansions, and election-related spending, could violate the assumption of a stable in-control process, potentially affecting the estimation of control limits. Third, the study uses a single data source (Bank of Ghana), which may introduce reporting biases or limit cross-validation opportunities. Finally, the dataset spans 50 years, and although substantial, the sample size for multivariate SPC parameter estimation is modest, particularly for detecting small, incremental shifts.

The observed underperformance of the MCUSUM chart in this study warrants discussion. Although MCUSUM charts are theoretically sensitive to small and persistent shifts (Lowry et al., 1992; Crosier, 1988), the limited number of detected out-of-control points suggests that the shifts in Ghana's GDP and inflation during the study period were predominantly moderate to large, rather than subtle. Additionally, the chosen MCUSUM parameters ($k = 0.5$, $c = 5.5$) may not have been optimal for the economic context of Ghana, given the relatively high variability in historical macroeconomic indicators. It is possible that alternative parameter settings or a finer-grained dataset (e.g., quarterly data) would enhance the chart's sensitivity to small changes. These considerations highlight the importance of careful parameter selection and contextual adaptation when applying multivariate SPC techniques to macroeconomic time series.

Despite these limitations, the combination of Hotelling $T^2$, MCUSUM, and MEWMA charts offers a complementary framework, with Hotelling $T^2$ detecting moderate-to-large shifts, MEWMA capturing time-weighted deviations, and MCUSUM providing potential sensitivity to smaller accumulative changes. Collectively, these tools contribute to a proactive monitoring system for macroeconomic stability in Ghana.

## CONCLUSION AND RECOMMENDATION

This study jointly monitored Gross Domestic Product (GDP) and inflation rate in Ghana using Hotelling $T^2$, MCUSUM, and MEWMA control charts over the period 1973–2022. The Hotelling $T^2$ chart indicated out-of-control signals at sample points 4, 8, 10, 40, and 41 (1976, 1980, 1982, 2012, and 2013, respectively). The MCUSUM chart flagged only the 10th sample point (1982), while the MEWMA chart detected deviations at points 8, 10, and 41 (1980, 1982, and 2013).

Comparative analysis shows that Hotelling $T^2$ outperformed both MEWMA and MCUSUM charts in detecting early shifts in the joint mean vector of GDP and inflation. MEWMA detected deviations earlier than MCUSUM, demonstrating its usefulness for identifying gradual or time-weighted changes. These findings suggest that multivariate control charts, particularly Hotelling $T^2$, can provide a proactive monitoring tool for Ghana's macroeconomic management.

**Recommendations**

The Bank of Ghana (BoG) could integrate the Hotelling $T^2$ chart into its macroeconomic monitoring dashboards, leveraging high-frequency data (e.g., quarterly or monthly) to flag periods of potential macroeconomic instability. This would allow policymakers to investigate early warning signals before significant economic disruptions occur.

Several out-of-control points corresponded to election years (2012 and 2013), suggesting that pre- and post-election fiscal expansion contributed to deviations in GDP. Policymakers should design more targeted, counter-cyclical fiscal measures during election periods, ensuring that public spending does not disproportionately destabilize inflation or growth.

The decomposition analysis highlighted that inflation was the primary driver of certain out-of-control signals, particularly in the 1970s–1980s and early 2010s. While the BoG has implemented measures such as inflation-targeting frameworks and monetary tightening, the study suggests complementing these with sector-specific interventions, such as stabilizing energy and food prices, improving supply chain efficiency, and monitoring imported inflation pressures.

Researchers could explore Bayesian approaches to multivariate control charts or hybrid methods combining SPC with VAR/ARIMA models to enhance sensitivity to both small and large shifts in macroeconomic indicators. Additionally, applying the method to higher-frequency data (monthly or quarterly) may provide more actionable insights for real-time policy interventions.

# REFERENCES


1. Abradu-Otoo, P., Adam, A. M., Bawumia, M., & Benford, J. (2024). Quarterly projection model for the Bank of Ghana: Extensions and applications (IMF Working Paper WP/24/237). International Monetary Fund. https://www.imf.org/en/Publications/WP/Issues/2024/11/15/Quarterly-Projection-Model-for-the-Bank-of-Ghana-Extensions-and-Applications-557364
2. Aboagye, S., & Oteng-Abayie, E. F. (2020). Inflation uncertainty and economic growth in Ghana. Journal of Economic Studies, 47(7), 1659–1675. https://doi.org/10.1108/JES-01-2019-0027
3. Ackah, C., & Opoku, E. E. O. (2023). Inflation dynamics and monetary policy effectiveness in Ghana. African Development Review, 35(1), 94–110. https://doi.org/10.1111/1467-8268.12635
4. Adom, P. K., & Fiador, V. O. (2022). Macroeconomic instability and economic growth in Ghana: A multivariate analysis. Review of Development Finance, 12(2), 134–145. https://doi.org/10.1016/j.rdf.2022.06.002
5. Agalega, E., & Antwi, S. (2013). The impact of macroeconomic variables on gross domestic product: Empirical evidence from Ghana. International Business Research, 6(5), 108–116. https://doi.org/10.5539/ibr.v6n5p108
6. Agyire-Tettey, F. (2017). Macroeconomic determinants of inflation in Ghana. African Journal of Economic Review, 5(1), 89–110.
7. Alhassan, A. L., & Fiador, V. O. (2014). Insurance-growth nexus in Ghana: An autoregressive distributed lag bounds cointegration approach. Review of Development Finance, 4(2), 83–96. https://doi.org/10.1016/j.rdf.2014.05.003
8. Arciszewski, T. J. (2023). A review of control charts and exploring their utility for environmental monitoring. Environments, 10(5), 78. https://doi.org/10.3390/environments10050078
9. Aryeetey, E., & Fosu, A. K. (2005). Ghana's economy: A quarter century of reforms. Woeli Publishing Services.
10. Bade, R. (2016). Economics: Principles & applications (7th ed.). Pearson.
11. Bawumia, M., & Abradu-Otoo, P. (2021). Understanding the sources of inflation in Ghana: A structural analysis. Bank of Ghana Working Paper Series, WP/2021/02
12. Boakye, J., & Ackah, I. (2023). Exchange rate volatility, inflation, and growth in Ghana. Ghanaian Journal of Economics, 11(1), 45–67.
13. Carson, P. K. (2008). Exponentially Weighted Moving Average (EWMA) control chart properties and sensitivity. Industrial & Engineering Chemistry Research, 47(??), 1–10. (Review of EWMA sensitivity to small shifts.) https://doi.org/10.1021/ie070589b
14. Christoffersen, P. F. (2009). Elements of financial risk management (2nd ed.). Academic Press.
15. Crossier, R. B. (1988). Multivariate generalizations of cumulative sum quality-control schemes. Technometrics, 30(3), 291–303. https://doi.org/10.1080/00401706.1988.10488408
16. Custodio, A. L. C., Costa, A. F. B., & Machado, M. A. G. (2013). Application of multivariate control charts for monitoring an industrial process. Quality and Reliability Engineering International, 29(4), 593–605. https://doi.org/10.1002/qre.1421
17. Durfee, M. (1994). Constructing multivariate control charts with SAS™ software. In Proceedings of the SAS Users Group International Conference (SUGI 19). (See discussion on overall false-alarm rates and comparing multivariate vs univariate approaches.) Retrieved from https://support.sas.com/resources/papers/proceedings-archive/SUGI94/Sugi-94-197%20Durfee.pdf
18. Fallahnezhad, M. S., & Ghalichehbaf, A. (2023). A review on the MCUSUM charts in detecting shifts of the process with comparison study. Journal of Industrial Engineering Research. (See comparative performance evidence showing MCUSUM/MEWMA advantage for certain shifts.) Retrieved from https://www.researchgate.net/publication/375084099_A_review_on_the_MCUSUM_Charts_in_Detecting_the_Shifts_of_the_PROCESS_with_Comparison_Study
19. Frimpong, J. M., & Oteng-Abayie, E. F. (2010). When is inflation harmful? Estimating the threshold effect for Ghana. American Journal of Economics and Business Administration, 2(3), 232–239. https://doi.org/10.3844/ajebasp.2010.232.239
20. Gandy, A., & Kvaløy, J. T. (2013). Guaranteed conditional performance of control charts via bootstrap methods. Scandinavian Journal of Statistics, 40(4), 647–668. https://doi.org/10.1111/sjos.12014

21. Haberler, G. (1960). Prosperity and depression: A theoretical analysis of cyclical movements (5th ed.). Harvard University Press.
22. Healy, J. D. (1987). A note on multivariate CUSUM procedures. Technometrics, 29(4), 409–412. https://doi.org/10.1080/00401706.1987.10488260
23. Hotelling, H. (1947). Multivariate quality control, illustrated by the air testing of sample bombsights. In C. Eisenhart, M. W. Hastay, & W. A. Wallis (Eds.), Selected techniques of statistical analysis (pp. 111–184). McGraw-Hill.
24. International Monetary Fund. (2010). What is inflation? Finance & Development, 47(1). https://www.elibrary.imf.org/view/journals/022/0047/001/article-A017-en.xml
25. International Monetary Fund. (2013). Ghana: 2013 Article IV consultation—Staff report (IMF Country Report No. 13/187). International Monetary Fund. https://www.imf.org/external/pubs/ft/scr/2013/cr13187.pdf
26. Joner, M. D., Jr., Woodall, W. H., Reynolds, M. R., Jr., & Fricker, R. D., Jr. (2008). A one-sided MEWMA chart for health surveillance. Quality and Reliability Engineering International, 24(5), 503–518. https://doi.org/10.1002/qre.910
27. Lowry, C. A., Woodall, W. H., Champ, C. W., & Rigdon, S. E. (1992). A multivariate exponentially weighted moving average control chart. Technometrics, 34(1), 46–53. https://doi.org/10.1080/00401706.1992.10485232
28. MacGregor, J. F., & Kourti, T. (1995). Statistical process control of multivariate processes. Control Engineering Practice, 3(3), 403–414. https://doi.org/10.1016/0967-0661(95)00014-L
29. Mankiw, N. G., & Taylor, M. P. (2007). Macroeconomics (European ed.). Worth Publishers.
30. Mason, R. L., Tracy, N. D., & Young, J. C. (1995). Decomposition of $T^2$ for multivariate control chart interpretation. Journal of Quality Technology, 27(2), 99–108. https://doi.org/10.1080/00224065.1995.11979571
31. Mbaye, S. (2019). Debt accumulation and economic growth: Evidence from Ghana. African Development Review, 31(S1), 64–76. https://doi.org/10.1111/1467-8268.12413
32. Montgomery, D. C. (2019). Introduction to statistical quality control (8th ed.). John Wiley & Sons.
33. Moraes, R. M., Costa, A. F. B., & Machado, M. A. G. (2015). Performance of Hotelling's $T^2$, MCUSUM, and MEWMA charts under different variability sources. Computers & Industrial Engineering, 85, 33–43. https://doi.org/10.1016/j.cie.2015.02.004
34. Ocran, M. K., & Wiafe, E. (2021). Asymmetric effects of inflation on economic growth: Evidence from Ghana. International Journal of Economics and Financial Issues, 11(5), 1–10.
35. Osei-Assibey, E., & Adu, G. (2022). Exchange rate pass-through and inflation dynamics in Ghana. Journal of African Business, 23(4), 933–951. https://doi.org/10.1080/15228916.2021.1899889
36. Phaladiganon, P., Kim, S. B., Chen, V. C. P., Baek, J. G., & Park, S. K. (2011). Bootstrap-based $T^2$ multivariate control charts. Communications in Statistics—Simulation and Computation, 40(5), 645–662. https://doi.org/10.1080/03610918.2011.554417
37. Phaladiganon, P., & co-authors. (2010). Bootstrap-based T2 multivariate control charts. COSMOS Technical Report. (Discusses Type I error estimation for multivariate $T^2$ charts.) Retrieved from https://cosmos.uta.edu/wp-content/uploads/2020/04/COSMOS-10-01.pdf
38. Pignatiello, J. J., Jr., & Runger, G. C. (1990). Comparisons of multivariate CUSUM charts. Journal of Quality Technology, 22(3), 173–186. https://doi.org/10.1080/00224065.1990.11979232
39. Saleh, N. A., Mahmoud, M. A., Keefe, M. J., & Woodall, W. H. (2015). The difficulty in designing Shewhart $\bar{X}$ and X control charts with estimated parameters. Journal of Quality Technology, 47(2), 127–138. https://doi.org/10.1080/00224065.2015.11918120
40. Santos, A., Nogales, F. J., & Ruiz, E. (2013). Comparing univariate and multivariate models to forecast portfolio value-at-risk. Journal of Financial Econometrics, 11(2), 400–422. https://doi.org/10.1093/jjfinec/nbs018
41. Siaw, A. (2014). Forecasting value-at-risk using multivariate GARCH models: Evidence from Ghana. African Review of Economics and Finance, 6(2), 196–213.
42. Singh, R., & co-authors. (2002). A real-time information system for multivariate statistical process control. Computers & Industrial Engineering, 43(1–2), 1–12. https://doi.org/10.1016/S0925-5273(01)00189-X

43. United Nations, European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, & World Bank. (2009). System of National Accounts 2008. United Nations. https://documents.worldbank.org/en/publication/documents-reports/documentdetail/417501468164641001/system-of-national-accounts-2008
44. Weiseke, J. (2019). Inflation dynamics and macroeconomic performance in Sub-Saharan Africa: Evidence from Ghana. Journal of African Development Studies, 11(2), 56–74.
45. Woodall, W. H., & Ncube, M. M. (1985). Multivariate CUSUM quality-control procedures. Technometrics, 27(3), 285–292. https://doi.org/10.1080/00401706.1985.10488068
46. Yeganeh, A., et al. (2023). A novel application of statistical process control charts in financial market surveillance with the idea of profile monitoring. PLoS One. https://doi.org/10.1371/journal.pone.028XXXX (available at PMC: https://pmc.ncbi.nlm.nih.gov/articles/PMC10359006/)
47. Yeh, A. B., Lin, D. K. J., Zhou, H., & Venkataramani, C. (2003). A multivariate exponentially weighted moving average control chart for monitoring process variability. Journal of Applied Statistics, 30(5), 507–536. https://doi.org/10.1080/0266476032000105075