Simulation of Multiferroic Hysteresis Loops Using Different Models

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ABSTRACT

Multiferroics show multiple ferroic ordering. These materials show multiple types relaxations and hysteresis loops when certain parameters are plotted. The present paper describes the models and hysteresis behavior of multiferroic materials. Ferroelectric, ferromagnetic, magneto electric and memristor hysteresis loops are simulated. Ginsburg Landau model is used for ferroelectric hysteresis, Jiles-Atherton model is used for ferromagnetic model. Joglekar model is used for memristor. Romberg Osgood model is used for ferroelastic hysteresis. The Models algorithm, input and output for the program are discussed

INTRODUCTION

The new magneto, electro, elastic memristor materials refer to a class of materials which exhibiting full coupling between mechanical, electric, elastic, and magnetic fields. These materials are called multi ferroic materials and show more than one type of hysteresis. There are few natural materials which possess simultaneous multi-ferroic coupling properties, composite materials composed of a piezoelectric and piezomagnetic phases have piezoelectric, piezomagnetic and magneto-electric coupling that is not present in the constituent phases. These materials show possibility of conversion of one form of energy into the other. Such materials have potential applications magneto-electric memory elements, smart sensors, and transducers as they can facilitate the conversion of one form of energy into the other [1-4]. The present paper reviews the ferroelectric, ferromagnetic, ferroelastic and memristor hysteresis property and the mathematical models. We also present the result of our simulations.

Constitutive equations:

All the theoretical investigations mentioned above are based on the following constitutive relations [5,6]

$$T_{ij} = C_{ijkl}^{E,H} S_{kl} - e_{kij}^{H} E_k - \bar{e}_{kij}^{E} H_k$$

$$D_i = e_{ikl}^{H} S_{kl} + \epsilon_{ik}^{S,H} E_m + \alpha_{ik}^{S} H_k \qquad (1)$$

$$B_i = \bar{e}_{ikl}^{E} S_{kl} + \alpha_{ik}^{S} E_m + \mu_{ik}^{S,E} H_k$$

Where T_{ij} is the stress tensor, D_i, is the electric displacement vector and $B_{i\ is\ the}$ magnetic flux density vector. The independent variables are the strains S_{ij}, the electric fields E_i, and the magnetic fields H_i. The thermodynamic potential associated with Equations (1) is

$$U_i = \frac{1}{2}(T_{ij}S_{ij} - E_i D_i - H_i B_i) \qquad (2)$$

The equations (1) can also be written as shown below by algebraically isolating the strain tensor (S_{ij}) and substituting it into the remaining equations for D_i and B_i. This shift from independent strains to independent

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stresses redefines the material coefficients from clamped states (constant strain, (S_{ij})) to free states (constant stress, (T_i))

$$S_{ij} = S_{ijkl}^{E,H} T_{ij} + d_{kij}^{H} E_k - \bar{d}_{kij}^{E} H_k$$

$$D_i = d_{ikl}^{H} T_{kl} + \epsilon_{ik}^{T,H} E_k + \alpha_{ik}^{T} H_k \qquad (3)$$

$$B_i = \bar{d}_{ikl}^{E} T_{kl} + \alpha_{ik}^{T} E_k + \mu_{ik}^{T,E} H_k$$

Definitions of various energy conversion coefficients that arise from the direct and converse constitutive equations are given in the table 1.

Table 1: Various coefficients can be defined using these equations [5]

S.No	Direct effect	Definition	Converse effect	definition
1	elasticity	$\frac{dT}{dS}$	Compliance	$\frac{dS}{dT}$
2	Direct piezoelectricity	$\frac{dT}{dE}$	Inverse piezo-electricty	$\frac{dD}{dS}$
3	Magneto-electric	$\frac{dD}{dH}$	Inverse magneto-electric effect	$\frac{dB}{dE}$

Ferroic Material Relaxations

All materials can be modeled as series or parallel combination of passive elements. Ferroelectric materials representation will have series /parallel combination of resistance and capacitors. Ferromagnetic materials will have series parallel combination of inductors and resistors. Ferro electromagnetic materials will have all these elements in both series and parallel combinations. All these show hysteresis of different parameters under suitable conditions (Table 2). Different models used to explain these properties also mentioned in the Table 2.Ferroelectric materials will have electric charge relaxations. Ferromagnetic materials will have magnetic pole relaxations. Ferro elastic materials are associated with elastic relaxations. The passive circuit elements which do not have sources of voltages and currents are:

$$\text{Resistor: } v = f(i) \;\Rightarrow\; v = Ri \;\Rightarrow\; dv = R\,di$$

$$\text{Capacitor: } q = f(v) \;\Rightarrow\; q = Cv \;\Rightarrow\; dq = C\,dv$$

$$\text{Inductor: } \varphi = f(i) \;\Rightarrow\; \varphi = Li \;\Rightarrow\; d\varphi = L\,di$$

$$\text{Mechanical damper: } \sigma = f(\varepsilon) \;\Rightarrow\; \sigma = K\varepsilon \;\Rightarrow\; d\sigma = K\,d\varepsilon$$

$$\text{Memristor: } \varphi = f(q) \;\Rightarrow\; \varphi = Mq \;\Rightarrow\; d\varphi = M\,dq \qquad (4)$$

Here v=voltage, i=current, R=resistance, q=charge, t=time, C=capacitance, L=inductor, φ=flux, M=memristor, σ =stress, ε=strain, K=elastic modulus. Memristor is fourth passive element [7,8]. Volt-ampere characteristics are different for these different circuit elements. For resistor it is linear. For other passive elements it is nonlinear. Ferroic materials when subjected to temperature variation, dT, electrical field, E, and stress, σ, magnetic filed H, will develop electrical charges, q

$$q \propto k_1 \delta T + k_2 E + K_3 \sigma + k_4 H + \ldots \quad (5)$$

K_1, k_2, k_3 and k_4 are pyro-electric, ferroelectric, elastic, magnetostrictive coefficients respectively. These coefficients are defined in tensor notation in table 1.

Multi ferroic (MF) materials exhibit two or more ferroic orders, such as magnetic: ferro-magnetic (FM), anti ferro-magnetic (AFM) or ferri-magnetic; ferro-electric (FE); ferro-elastic; or ferro-toroidic [1,2]. Magneto-electic (ME) materials, on the other hand, exhibit coupling between the electric and magnetic degrees of freedom so that, in these materials, an electric (magnetic) polarization can be induced by a magnetic (electric) field. Although there are MF materials that are not ME and vice versa, for fundamental reasons, the ME coupling in single-phase materials is largest in MF materials. Magneto-electric multi ferroics allow the possibility of switching the magnetization with electric field [10,11]. This offers a wealth of opportunity for information storage applications. In particular, it could remove the main hindrance in the miniaturization of magnetic random access memory (MRAM), where the write operation requires magnetic fields or large currents [12]. An electric field (E) produces an electric dipole moment and hence electric polarization (P) in the material. Conversely, a magnetic field (H) produces magnetization (M). Stress (s) produces strain. The electro-magnetic source vector (S) produces toroidal moment (t). For small fields, the relationship is linear, but nonlinear effects become important for larger fields.

Macroscopically, the interaction of the electric (E) and magnetic (H) fields and the stress tensor (s) with the material can be described within the Landau theory of phase transitions [12]. The expansion of the free energy (U) up to second order in the fields in the SI system of units can be written as

$$U = U_0 - P_S E - \mu_0 M_S - \epsilon_s \sigma - \frac{1}{2}\epsilon_0 \chi_E E^2 - \frac{1}{2} s\sigma^2 - \frac{1}{2}\mu_0 \chi_M H^2 - \alpha EH - dE\sigma - qH\sigma + \cdots \quad (6)$$

where an implicit summation over the vector and tensor indices is understood. The first term is the free energy at zero fields (U_0). The next three first-order terms represent the dipole energies related to the spontaneous electric polarization (P_S), magnetization (M_S) and elastic deformation (ε_s) interacting with the conjugated fields. The next three second-order terms reflect the electrostatic, magneto-static and elastic energy in materials with electric susceptibility (χ_E), magnetic susceptibility (χ_M) and elastic tensor (s), respectively. The remaining second order terms represent the linear ME (α), piezoelectric (d) and piezomagnetic (q) effects. Higher order terms involving electrostriction (σE^2, $E\sigma^2$), Magnetostriction (σH^2, $H\sigma^2$) and nonlinear magnetoelectricity (HE^2, EH^2) are not included in equation (6) for simplicity. Taking derivatives of the free energy with respect to the fields, we can obtain the macroscopic order parameters

$$P = -\frac{\partial U}{\partial E} = P_S + \epsilon_o \chi_E E + \alpha H + d\sigma + \cdots.$$

$$\mu_0 M = -\frac{\partial U}{\partial H} = \mu_0 M_S + \mu_o \chi_M H + \alpha E + q\sigma + \cdots.$$

$$\varepsilon = -\frac{\partial U}{\partial \sigma} = \epsilon_S + s\sigma + dE + qH + \cdots. \quad (7)$$

Ferroelectric materials are analogous to ferro-magnetics, using electric fields instead of magnetic fields to orient dipoles. Ferroelectric, ferromagnetic and ferroelastic properties are realized is both single phase and composite material. Memristic property is different and could only be realized in metal nano films only. Various models that describe different hysteresis phenomena are given in table 2.

Table 2: Theories of ferroic hysteresis phenomena

S. No	material	Model	Hysteresis
1	Ferroelectric	Ginsburg Landau [6]	DE loop

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2	Ferromagnetic	Ginsburg Landau [6] Jiles Atherton Model [14,15]	BH Loop
3	Ferroelastic	Ramberg Osgood [13]	Stress strain loop
4	Memrister	Joglekar Model [17,18]	VI loop

Ginsburg Landau Model

Ferroelectric and ferromagnetic system's equilibrium behavior near the phase transition is explained by Landau theory [6]. A ferroelectric/ferromagnetic system changes from ferro-electric /ferro-magnetic and para-electric/para-magnetic phases due to change in symmetry. Landau theory uses an order parameterto characterize the transition of higher symmetry to lower symmetry. The polarization (P) is the order parameter for para-electric/para-magnetic/ferro-electric/ferro-magnetic phase transition. The free energy U(P) goes to minimum for the equilibrium state of the system. The free energy U is expressed as a power series of order parameter inthe ferroelectric/ferromagneticphasetransition [1,2]. The polarization/magnetization hysteretic behavior of ferro-electric/ferro-magnetic is derived from the Landau-Devonshire expression for freeenergy U(P) in terms of polarization which is given as [6],

$$U(P) = U_o - \frac{\alpha_1}{2}P^2 + \frac{\alpha_2}{4}P^4 + \ldots \quad (8)$$

where α_1 and α_2 are Landau coefficients and U_0 is the free energy in para state. Applying the minima and maxima conditions by taking the derivative of above equation and neglecting the higher order terms, we have [6].

$$\frac{\partial U}{\partial P} = E = -\alpha_1 P + \alpha_2 P^3 \quad (9)$$

$$\alpha_2 P^3 - \alpha P - E = 0 \quad (10)$$

The roots of this equation gives value of spontaneous polarization/magnetization and the value of field (*E or H)*at local minimum and maximum gives the value of coercive field. Landau coefficients α_1 and α_2 can be calculated from the experimental measured values of spontaneous polarization/magnetization (P_s of M_s) and coercive field (E_c or H_c) using:

$$\alpha_1 = \alpha_2 P_s^2 \text{ and } \alpha_2 = \frac{3\sqrt{3}E_C}{2P_S^3} \quad (11)$$

The value of polarization for each value of electric field (*E*) is calculated by finding the cubic roots of polarization equation are P_1, P_2, and P_3 :

$$P_1(E) = S + T$$

$$P_2(E) = -\frac{1}{2}(S+T) + j\frac{1}{2}\sqrt{3}(S-T)$$

$$P_3(E) = -\tfrac{1}{2}(S+T) - j\tfrac{1}{2}\sqrt{3}(S-T) \quad (12)$$

$$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}} \qquad T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}$$

$$R = \frac{E}{2\beta} \quad Q = -\frac{\alpha}{3\beta} \quad (13)$$

Table 3: Input parameters for ferroelectric hysteresis loop simulation

Parameter	Value
E_C	0.4847×10^6 V/cm

ISSNNo.2454-6194 | DOI:10.51584/IJRIAS | Volume XI Issue VI June 2026

P_S	$6.8X10^{-6}$ C/cm^2
α_1	1.85 x10^{11} cm/F
α_2	4x10^{21}cm^2/FC^2

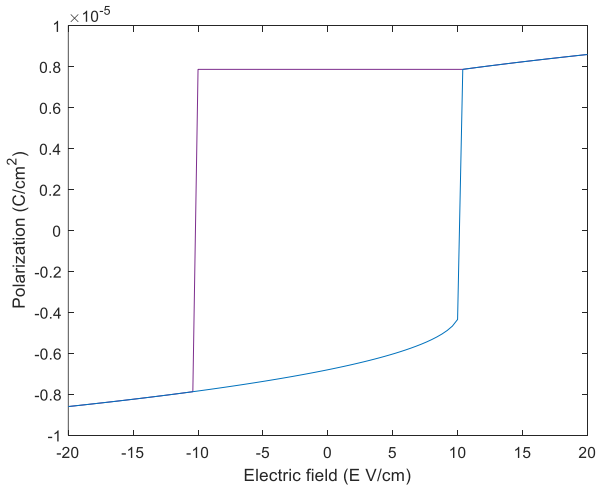

Figure 1. Ferroelectric hysteresis loop

Algorithm-Ferroelectric Hysteresis (P-E):
Input: Electric field E(t), initial polarization P0, time step Δt
Output: Ferroelectric hysteresis loop P-E
1. Initialize P(1) ← P0, set H(t) ← 0 and σ(t) ← 0.
2. For each time step i, compute roots of equation 10.
3. Update polarization using P(i)
4. Store E(i) and P(i).
5. Plot P versus E

Figure 1 gives the ferro-electric hysteresis loop obtained from simulation. In this paper the simulation has started with equation (8) for Free Energy (U) which when differentiated with reference to Electric field (E) results in cubic equation for Polarization (P) equation (10). This cubic equation when solved gives a hysteresis loop.Ginsburg parameters α_1 and α_2 (Table 3) assumed for this simulation are as 1.85 x 10^{11}cm/F and 4 x 10^{21} cm^2/FC^{2}respectively. Coercive field and spontaneous polarization values are assumed to be 0.4847 x 10^6 V/cm and $6.8X10^{-6}$ C/cm^2 respectively which correspond to Poly (vinylidene fluoride trifluoroethylene)[P(VDF-TrFE)] material. The P values (roots of cubic equation) obtained are acceptable if and only if the root values lie within the Polarization [$-P_s$, P_s] and Electric field [$-E_c$, E_c] intervals.

Jiles-Atherton (JA) model

The Jiles-Atherton (JA) model [14,15] is a physics-based approach used to simulate magnetic hysteresis, calculating magnetization M in response to an applied magnetic field H. Five equations are introduced in J-A model to describe the relationship between the external magnetic field *H*, effective magnetic field *H*e, anhysteretic magnetization *M*an, irreversible magnetization M_{irr}, reversible magnetization M_{rev} and total magnetization *M*. Commonly recognized, the model takes modified Langevin equation to fit the anhysteretic magnetization and deduces the expression of the hysteresis from the equation of Energy Conservation. Though the based mechanism is the same, the expressions of J-A model are not uniformed in many references.

The differential equation describing the Jiles Atherton model is given by:

$$\frac{dM}{dH} = \frac{1}{(1+c)} \frac{(M_{an}-M)}{\frac{\delta k}{\mu_0}-\alpha(M_{an}-M)} + \frac{c}{(1+c)} \frac{dM_{an}}{dH} \qquad (14)$$

ISSNNo.2454-6194 | DOI:10.51584/IJRIAS | Volume XI Issue VI June 2026

The anhysteretic magnetization M_{an} is given by Eq. (12) using the Langevin function:

$$M_{an} = M_S\left[\coth\left(\frac{H+\alpha M}{a}\right) - \frac{a}{H+\alpha M}\right] \qquad (15)$$

Where M is the total magnetization, H is the applied magnetic field, δ=sgn(dH/dt), t is the time, kis the magnetic permeability of free space and saturation magnetizationM_S , domain flexing a, inter domain coupling α, component ratio c, domain wall pinning k, are the JA parameters.

Although the Langevin function is not singular, its evaluation for small values of the argument is difficult, since the computer has to subtract two functions that are singular at the origin. One way to overcome this problem is to replace the Langevin function by its series expansion. Another possibility, which in fact we have chosen in order to improve computation speed, is to replace the Langevin function by an equivalent look-up table, which can be generated either through the mentioned expansion or directly.

The differential equation (14), which in its original form has derivatives with respect to H, was reformulated into a differential equation in time by multiplying the left and the right sides by dH/dt, thus resulting in

$$\frac{dM}{dt} = \frac{1}{(1+c)}\frac{dH}{dT}\frac{(M_{an}-M)}{\frac{\delta k}{\mu_0}-\alpha(M_{an}-M)} + \frac{c}{(1+c)}\frac{dM_{an}}{dt} \qquad (16)$$

For a given set of values of the material nickel, the JA parameters (M_S, a, a, c, k), a simulation (Table 4) is conducted and the calculatedpairs (B, H) are collected with an appropriate sampling rate so as to allow the comparison with the experimentaldata

Table 4: Jiles-Atherton (JA) model Parameters used for simulating Magnetic hysteresis

C	Parameter
1	a = 30
2	c = 0.95
3	k /μ_0= 10
4	M_s = 380X10^3
4	l_s = 10*10^-6;
5	α= 10^{-6}

Algorithm Ferromagnetic Hysteresis

Input: Magnetic field H(t), initial magnetization M0, time step Δt

Output: Ferromagnetic hysteresis loop M-H or B-H

1. Initialize M (1) ← M0, set E(t) ← 0 and σ(t) ← 0.
2. For each time step solve the equations 14 and 16
3. Update magnetization M(i).
4. Compute magnetic flux density B(i) ← H(i) + M(i).
5. Plot M versus H or B versus H.

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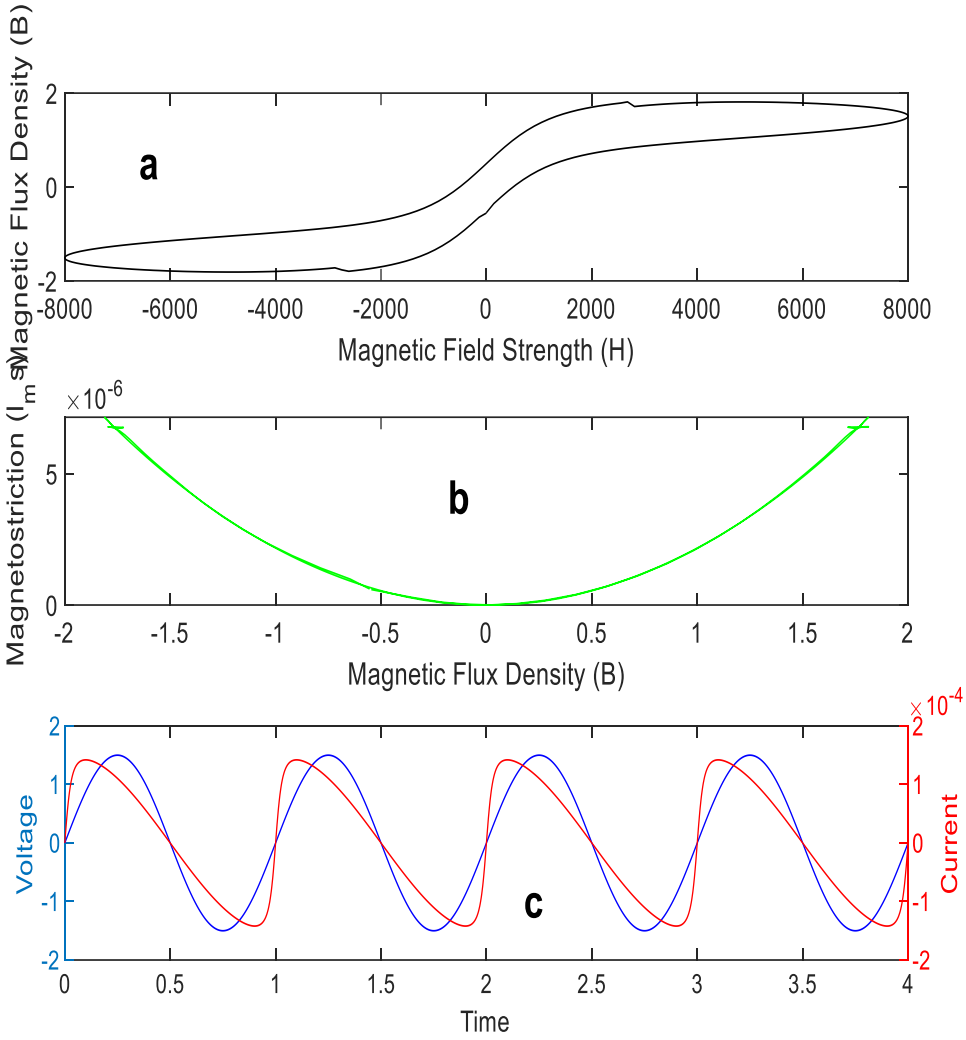

Figure2 a. Ferromagnetic hysteresis
b. Magnetostriction as a function of flux density
c. variation of V and I as a function of time t

Figure 2 gives the ferroelectric hysteresis loop obtained from simulation. Figure 2a gives Ferromagnetic hysteresis loop. Figure 2b gives the calculated Magnetostriction as a function of flux density. Figure2c Variation of V and I as a function of time t which reflects the phase difference between these parameters. Magnetic hysteresis can also be explained using the equations 8,9, and, 10 by replacing the polarization (P) with Magnetization (M), Electric field (E) with Magnetic Field (H) and Landau parameters corresponding to magnetic phenomena. However, in the present simulation we have used J-A model described by equations 14-16. We have used Nickel, saturation magnetization, interdomain coupling α, domain anhysteric constant (a) and domain pinning constant (K) for the purpose of simulation of J-A model is expected to give better hysteresis loop closer to the experimental values.

Ferro elastic Model

Ferroelastic materials are ferroic materials in which the primary order parameter is spontaneous strain [13]. Similar to ferroelectric materials, where polarization can be switched by an electric field, and ferromagnetic materials, where magnetization can be switched by a magnetic field, ferroelastic materials exhibit strain states that can be switched by applied mechanical stress. When stress is applied, the material response is not purely Hookean; instead, the stress–strain curve becomes nonlinear and shows hysteresis due to domain switching, twin-boundary motion, and delayed strain recovery. Ferroelastic materials show a highly non-Hookean strain–stress hysteresis response when mechanical stress is applied.

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In ferroelastic systems, different orientation variants of the crystal structure can exist. Under applied external stress, the elastic energy of the system shows relaxation. During unloading, the material does not immediately return through the same path because some domain walls remain pinned or relax gradually. This produces a closed stress–strain loopwhere σ is the applied stress and ϵ is the strain. The area enclosed by the loop represents mechanical energy dissipated during one loading cycle:

$$W = \oint \sigma \, d\epsilon \qquad (17)$$

This behavior is analogous to the energy loss observed in ferroelectric $P - E$ and ferromagnetic $B - H$ hysteresis loops.

Ramberg–Osgood Model for Ferroelastic Hysteresis

The nonlinear stress–strain response is modelled using the Ramberg–Osgood relation [13]. This model is commonly used to represent smooth nonlinear monotonic and cyclic stress–strain curves, especially when there is no sharp yield point. The total strain is written as the sum of elastic strain and nonlinear strain:

$$\epsilon = \epsilon_e + \epsilon_{nl} \qquad (18)$$

Where $\epsilon_e = \frac{\sigma}{E}$ and $\epsilon_{nl} = \left(\frac{\sigma}{K'}\right)^{1/n'}$

Therefore, the Ramberg–Osgood equation is:

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} \qquad (19)$$

Where E is Young's modulus, K' is the cyclic strength coefficient, and n' is the cyclic strain hardening exponent.For cyclic loading, unloading and reloading are generated using Masing's rule. The Masing approach is widely used with the Ramberg–Osgood relation for cyclic stress–strain behaviour.

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \qquad (20)$$

where $\Delta\sigma$ is the change in stress during unloading or reloading and $\Delta\epsilon$ is the corresponding change in strain.

Algorithm-Ferroelastic Hysteresis $(\epsilon - \sigma)$

Input: Applied stress $\sigma(t)$, initial strain state, time step Δt

Output: Ferroelastic stress–strain hysteresis loop

1. Define cyclic stress input $\sigma(t) = \sigma_0 \sin(\omega t)$.
2. Compute strain using Ramberg–Osgood relation: $\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$.
3. For unloading and reloading, apply Masing's rule: $\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$.
4. Store loading, unloading, and reloading values of σ and ϵ.
5. Plot ϵ versus $\sigma$and calculate energy loss ($\oint \sigma \, d\epsilon$).

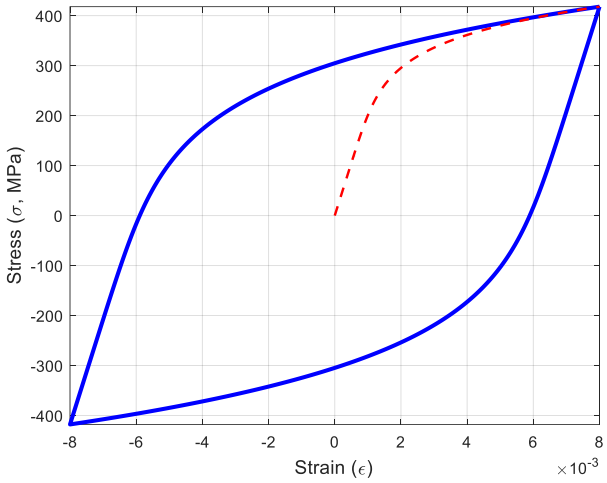

Figure 3. Elastic hysteresis plot

Figure 3 shows the Elastic hysteresis plot obtained from the present simulation. We have simulated elastic hysteresis loop using young's modulus $2.1x10^5$ MPA corresponding to steel. The value of K′ = 100 and N′=0.015 these are the parameters of equation 19. The area within the hysteresis loop gives the internal friction of the material.

Memristor Model

The memristor can be realized using TiO_2 nano particles [7,8]. A part of it is doped and the other part is undoped. The equivalent passive element representation is as shown in the figure 4 using resistors

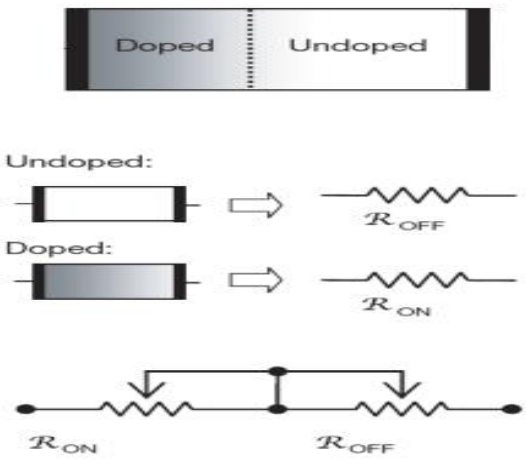

Figure 4: Memristor schematic

Memristor can be defined as the ratio between time rate of change of magnetic flux (voltage) to time rate of change of charge (current). Hence memristor will have dimensions of resistance [18]:

$$\boldsymbol{M(q(t)} = \frac{\frac{d\emptyset}{dt}}{\frac{dq}{dt}} = \frac{\boldsymbol{V(t)}}{\boldsymbol{I(t)}} \quad (21)$$

Each memristor model is based on two equations, organized in a system. The first expression relates the memristor voltage and current. This equation contains the state variable. The second equation connects the time derivative of state variable x and the memristor current i_{mem} (or the memristor voltage v_{mem}). The Joglekar memristor model [18] is

$$v_{mem} = [R_{ON}x + R_{OFF}(1-x)]i_{mem}$$

$$\frac{dx}{dt} = \eta k i_{mem}(v_{mem}) f(x); x_{t=0} = x_0 \quad (22)$$

Here, the ON-state and OFF-state resistances, usually known as memristance, are denoted as RON and ROFF. The quantity p is a positive integer. When increasing its value, the Joglekar window function f(x) becomes flatter in the central region and the equations are:

$$v_{mem} = [R_{ON}x + R_{OFF}(1-x)]i_{mem}$$

$$\frac{dx}{dt} = \eta k i_{mem}(v_{mem})[1-(2x-1)^{2p}; x_{t=0} = x_0 \quad (23)$$

Algorithm Memristor Hysteresis (V, x0, Δt)
Input: Applied voltage V(t), initial state x0, time step Δt
Output: Pinched memristor hysteresis loop I-V
1. Initialize x(1) ← x0, where 0 ≤ x ≤ 1.
2. For each time step i, compute R(i) ← RON x(i) + ROFF[1 − x(i)].
3. Compute current I(i) ← V(i) / R(i).
4. Update state x(i+1) ← x(i) + ηkI(i)[1 − (2x(i) − 1)^(2p)]Δt.
5. Plot I versus V.

Figure 5 shows the results for the present simulation. Figure 5a shows variation of V and I as a function of time. Figure 5b. Resistance versus time plot of memristor. Figure 5c shows the voltage and current (VI) hysteresis of memristor. Figure 5d shows the R-V characteristics of memristor.

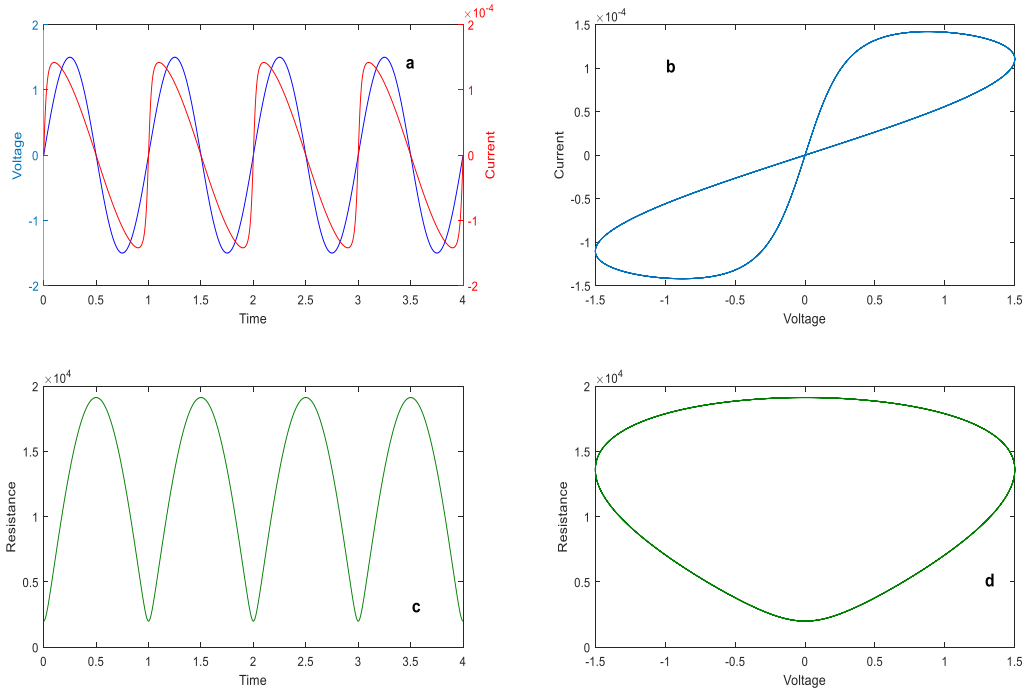

Figure 5.a. variation of V and I as a function of tile
b. VI hysteresis of memristor
c. Resistance versus time plot of memristor
d. Memristor resistance–voltage plot

The values used for the simulation are: R_{on}= 100Ω, R_{off}= 200 kΩ and, Width W= 1.5 nm. The memristor is simulated using thin films of doped TiO_2.

CONCLUSIONS

Ferro-electric / Ferro-magnetic: These rely on a non-equilibrium minimization of the Landau double-well free energy landscape, where α_1, $\beta_1 < 0$. The delay in the order parameters (P, M) relaxing into the moving global minimum creates the classic wide hysteresis opening. Ferro-magnetic materials also use JA model. Memristor produces a distinctive pinched hysteresis loop crossing exactly through the origin. Because it does not store net charge or flux statically like a capacitor or inductor, the current I must dynamically fall to zero whenever voltage V = 0, satisfying I(t) = V(t)/R(w). Ferro-elastic hysteresis is because of phase lag between stress and strain and internal friction. It may not be prudent to simulate multi-ferroics including memristors as these are totally different class of devices of very low dimensions.

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