

Mathematical Modeling of Unsteady MHD Convective Flow through a Porous Medium with Chemical Reactions

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ABSTRACT

The unsteady magnetohydrodynamic (MHD) free convective oscillating flow of an optically thin incompressible viscous fluid embedded in two parallel porous walls under the influence of an externally applied transverse magnetic field with a chemical reaction. Rotating disks involving chemical reactions find extensive applications in the medical field, particularly in optimizing chemical processes for controlled drug delivery systems, thereby improving precision in therapeutic treatments. In this study, we elucidate the behavior of fluid flow over a rotating disk with Casson ternary hybrid nanofluid with incorporation of chemical reactions. Subsequently, the bvp4c solver in MATLAB is utilized to numerically solve the set of non-linear ordinary differential equations describing the boundary value problem. Overall, these results offer valuable insights into optimizing heat and mass transfer in engineering systems involving non-Newtonian nanofluids under chemical reaction environments.

Key words: MHD, Porous medium, Chemical reaction, Heat transfer, Mass transfer

INTRODUCTION

The MHD free convective flows occurred recurrently in natural world. The functions of MHD incompressible viscous flows in sciences and engineering connecting heat transport under the influence of chemical reacting are of enormous significance to lot of regions of science and engineering streams. These commonly occurred in the petro-chemical industries, power and cooling system, chemical vapor deposition on surfaces, freezing of nuclear reactor, temperature exchangers designing, forest firing mechanics, geophysical and MHD power generations system. The systematic solutions for the unsteady free central heating flow through porous medium and had been examined by Magyari et al.[1] Hossain et al.[2] explored the effects on MHD mixed heat and mass transport on natural central heating commence porous surface fixed into a fluid saturate porous medium. Mazumdar and Deka [3] discussed the MHD flow past the impulsively enchanting consign of an infinitely vertical plate in the occurrences of thermal radiation. Sharma et al.[4] and Mahapatra et al.[5] examined the consequences of chemical reacting for free convection flow past a porous medium enclosed over the perpendicular surface. Rajasekhara et al.,[6] Kishan and Srinivas,[7] Anjalidevi and David [8] explored the consequences of an assortment of variables on fluid flow quantity.

Rotational fluid dynamics in industrial applications is a vast field that encompasses various studies and practical implications. The importance of these dynamics extends to medical applications, such as blood flow within spinning devices like artificial hearts or centrifugation operations in diagnostic laboratories [9]. Moreover, the application of CFD in studying the effects of Casson fluid, a non-Newtonian fluid model often used to represent blood, has been a subject of research, particularly in the context of chemical reactions and oscillating flow effects [10]. Casson fluid is particularly valuable for modeling non-Newtonian fluids such as blood due to its precision in representing blood's rheological properties essential for comprehensive studies on blood flow in diverse conditions and diseases [11,12,13]. In the medical field, understanding homogeneous and heterogeneous reactions in Casson fluid is crucial for modeling biological processes where reactions are distributed uniformly, such as chemical reactions within the bloodstream or cellular activities. Thus, the present study explicates transport phenomena over a rotating disk using a novel non-Newtonian fluid called Casson fluid paired with homogeneous and heterogeneous reactions.

Sahoo and Nand-keolyar [14], Ragupathi et al. [15], Ahmed et al. [16], Ramzan et al. [17] have shed light on key outcomes that focus on Casson fluid flow with chemical processes. The study's conclusion highlights pronounced effectiveness of homogeneous and heterogeneous chemical reaction parameters in diminishing concentration distribution and alter the fluid's properties, such as viscosity and thermal conductivity affecting the velocity and temperature profiles. These papers collectively demonstrate that the behavior of Casson nanofluids in rotating disk systems is highly sensitive to autocatalytic, thermal, and rotational effects. These insights are essential for advanced thermal management and bioengineering applications. Evidently, homogeneous and heterogeneous reactions occurring in the fluid flow over rotating disks have the potential to enhance system entropy optimization, thereby contributing to increased efficiency and lowered energy consumption in diverse industrial processes.

In addition to being involved in a number of issues, there are a lot of significant profits. Many materials are utilized for machinery, hydro-energy, medicine equipments, and states of the arts facility because of those capabilities. The physical applications of atmospheric substances and engineering promote the study of changes in the oceanographically science. As well as, its applications have been grown into the food industries, chemical engineering, filtrations system, and machinery science. In the situation at speed and velocities specific to the earth which will rotate in relation to each axis. Knowledge of the landscape and the latter have been become an essential part since of those interests and commitments to those surrounding. As a further study of astronomy is concerning, the study of physical science has become progressively more imperative as it caught be studied in fluids and the solar material, intermolecular and intra-molecular solar beams, etc. These detect bias MHD generators and electrical motors for input and outputs. Here, they have some significant engineering and geological features of the canal from the planets, water study and waterways, and to study usual gases, oils and water situations by oils field and oils technology. The practice of diving in the last dungeon is exciting to both science and technology to find out that reveals the correct kinds of situations are a most important and difficult for resolving fragmentary water problems. Despite the revelations expressed by most, here is the immense deal of evidences that is not tolerates depending on those subjects matter and the other critical issues. Through everyone on the top of study, the united consequence of radiation and chemical reaction on the MHD convective heat transfer flow had not been considered as at the similar time. A moment ago, Krishna and Chamkha,[18] Krishna et al.,[19] and Krishna and Chamkha[20] explored the MHD flow through porous medium in planar ducts/channels. The influence of thermal radiating, Hall and ions slip consequences on an unsteady MHD free convection revolving flow of Jeffrey liquids over the unbounded perpendicular porous plate by the ramped wall temperature have been explored by Krishna.[21] Krishna[22] discussed the radiating absorption, chemically reacting consequences on MHD combined convection flows past the partially unbounded moving porous surfaces.

Mathematical formulation of the problem

We considered the unsteady MHD free convective oscillating flow of an optically thin incompressible viscous fluid embedded in two parallel porous walls under the influence of an external applied transverse magnetic field with chemical reaction, radiative heat flux, and heat absorption. The physical modeling of the investigation is displayed in Figure I. This is assumed that the fluid has tiny electrical conductivity and the electromagnetic forces created and is too small, so that it is neglected the induced magnetic field.

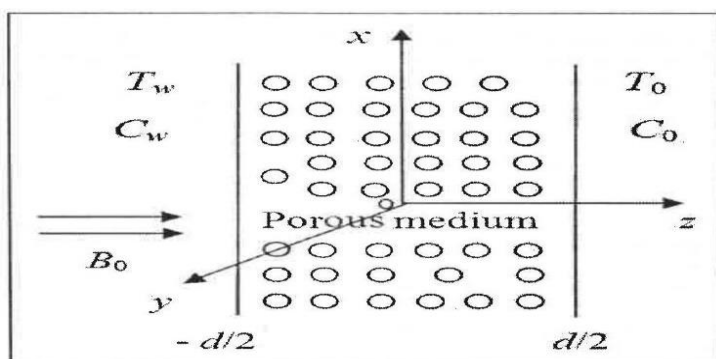


Figure I. Physical configuration of the problem.

$$\frac{\partial w}{\partial z} = 0, \frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{k} u + g\beta(T - T_0) + g\beta^*(C - C_0) \frac{\partial v}{\partial t} + w \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2 v}{\rho} - \frac{\nu}{k} v \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \frac{k_1}{\rho C_p} \frac{\partial^2 T}{\partial z^2} + \frac{Q_0 T}{\rho C_p} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial z} \frac{\partial C}{\partial t} + w \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial z^2} - KrC \quad (1-5)$$

The initial and boundary conditions are

$$u = 0, v = 0, T = 0, C = 0 \text{ for all } z \text{ and } t \leq 0 \quad u = 0, v = 0, T = 0, C = 0 \text{ at } z = -\frac{d}{2} \quad u = 0, v = 0, T = T_0 + (T_w - T_0)\cos \omega t, C = C_0 + (C_w - C_0)\cos \omega t \text{ at } z = \frac{d}{2} \quad (6-8)$$

Combining the equations (2) - (3), let $q = u + iv$, it is achieved as,

$$\frac{\partial q}{\partial t} + w \frac{\partial q}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 q}{\partial z^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k} \right) q + g\beta(T - T_0) + g\beta^*(C - C_0) \quad (9)$$

Consequently, this is contained that, the fluid is optically thick by comparatively minute density. The radiative heat flux is given by

$$\frac{\partial q_r}{\partial z} = 4x^2(T - T_0) \quad (10)$$

Where α is the radiating absorption coefficient.

The periodical pressure gradient as,

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = P \cos \omega t \quad (11)$$

Integrating the equation of continuity (1), it is obtained as,

$$w' = w_0 \quad (12)$$

We are introducing the non-dimensional variables as

$$x^* = \frac{x}{d}, z^* = \frac{z}{d}, \xi^* = \frac{\xi}{d}, q^* = \frac{q}{w_0}, \theta = \frac{T - T_0}{T_w - T_0}, \phi = \frac{C - C_0}{C_w - C_0}, t^* = \omega t, \omega^* = \frac{\omega d^2}{\nu}, p^* = \frac{p}{\rho w_0^2}.$$

Substituting the non-dimensional variables in the equations (4), (5), and (9), we get

$$\omega \frac{\partial q}{\partial t} + \lambda \frac{\partial q}{\partial z} = -\lambda \frac{\partial p}{\partial x} + \frac{\partial^2 q}{\partial z^2} - \left(M^2 + \frac{1}{K} \right) q + Gr \theta + Gm \phi \quad \omega \frac{\partial \theta}{\partial t} + \lambda \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} - \frac{1}{Pr} (N^2 - Q) \theta \quad \omega \frac{\partial \phi}{\partial t} + \lambda \frac{\partial \phi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial z^2} - Kc \phi \quad (13-15)$$

The initial and boundary conditions are,

$$u = 0, v = 0, \theta = 0, \phi = 0 \text{ for all } z \text{ and } t \leq 0 \quad (16)$$

$$u = 0, v = 0, \theta = 0, \phi = 0, \text{ for } t > 0, \text{ at } z = -\frac{1}{2} \quad u = 0, v = 0, \theta = \cos t, \phi = \cos t, \text{ for } t > 0 \text{ at } z = \frac{1}{2} \quad (17-18)$$

"Where, $M^2 = \frac{\sigma B_0^2 d^2}{\mu}$ Hartmann number, $K = \frac{k}{d^2}$ permeability parameter, $\lambda = \frac{w_0 d}{\nu}$ suction parameter, $Gr = \frac{g\beta d^2 (T - T_0)}{\nu w_0}$ thermal Grashofs number, $Gm = \frac{g\beta^* d^2 (C - C_0)}{\nu w_0}$ mass Grashofs number, $Kc = \frac{d^2 Kr}{\nu}$ chemical reaction parameter, $Sc = \frac{\nu}{D}$ Schmidt number, $Q = \frac{Q_0 d^2}{k_1}$ heat absorption parameter, $Pr = \frac{\mu C_p}{k_1}$ Prandtl number, $N = \frac{2\alpha d}{\sqrt{k_1}}$

radiation parameter." The system of Eqns. (13) to (15) together with initially along with frontier conditions (16)-(18) are resolved by utilizing Laplace Transform technique. The transformed solutions are

$$\begin{aligned} q^- &= a_{25}e^{m_5z} + a_{24}e^{m_6z} + a_{13} + a_{14}e^{m_3z} + a_{15}e^{m_4z} + a_{16}e^{m_1z} + a_{17}e^{m_1z} \quad \theta^- = a_9 \left(\frac{s}{s^2+1} \right) e^{m_3z} + \\ a_{10} \left(\frac{s}{s^2+1} \right) e^{m_4z} \quad \phi^- &= a_{11} \left(\frac{s}{s^2+1} \right) e^{m_1z} + a_{12} \left(\frac{s}{s^2+1} \right) e^{m_1z} \end{aligned} \quad (19-21)$$

"Implement the inverse Laplace transformations for the equations (19), (20) and (21), happens to unwieldy as of the existence of both the multiple poles and branch points. These integrals are willing to the solutions by means of asymptotic approximations. It is obtained the velocity, temperature and concentration distributions respectively. For engineering interest, it is obtained that the skinfriction, Nusselts number and Sherwood number near the left side plate as well as right side plate as,"

$$\tau = \left(\frac{\partial q}{\partial z} \right), Nu = \left(\frac{\partial \theta}{\partial z} \right), \text{ and } Sh = \left(\frac{\partial \phi}{\partial z} \right) \quad (22)$$

RESULTS AND DISCUSSION

The heat generating and/or absorption as well as thermo-diffusion on the unsteady free convection MHD gyrating flow of radiation and chemical reactive second order fluid past an unbounded perpendicular plate during absorbent medium have been discussed. The analytical solutions for the governing equations are found by utilization of Laplace transformation methodology. The velocity, temperature and concentration profiles are analyzed with some graphical profiles. It is represented the , temperature and concentration distributions for quite a few quantities of permeability parameter K, magnetic field parameter M, chemical reaction parameter Kr, are described in Figs. 1–7. For computational purpose we are fixing M = 0.5, K = 0.5, Pr = 0.71, R = 2, Gr = 10, Gm = 5, Sr = 0.1, H = 2, $\alpha = 0.5$, t = 0.5 and drawn the profiles being each parameter varied over the range. From Figs. 1–7, it is depicted that, concentration, temperature and velocity profiles are lessen in case of ramped wall temperature with ramped surface concentration than that of isothermal temperature with ramped surface concentration. Figs. 1 and 2 has been shown that, the intensity of the magnetic field has reducing effects on velocity profiles for together heated circumstances. It is anticipated to the information that, the representing of magnetic domain parameter produces electrical field in the liquid flow.

This implied that, the magnetic field has reducing effect for together ramped wall temperature with ramped surface concentration as well as isothermal temperature with ramped surface concentration. It is expected to the information that, the application of the magnetic field to fluid give augment to the resistive model forces (Lorentz forces) on the fluid in the boundary layers, this slow down the movement of the fluid. From Figs. 3 and 4, it is depicted that the variation of the velocity component with respected to the permeability parameter K. Mutually the velocity components u and v are enhancing by an escalating into the permeability parameters K. Therefore, an augmentation in the permeability of porous medium lead to mounts throughout the fluid flow region. Inferior the permeability lower the fluid velocity is displayed in entire liquid expanse. These results are also used in petroleum extraction process from crude oil in petroleum engineering. The thicknesses of the boundary layers increase when the permeability parameter increases throughout the fluid region. together thermal case as displayed into Fig. 5 as well as 6. It has been shown that, the destructive reactions $Kr > 0$ led to falls in the velocity field this in turn deteriorates the buoyancy impacts owing into the concentration gradient. Subsequently, the flow field is retarding. These incidences have the higher conformity by the physical reality. The chemical reactions have a reducing influences on liquid flow velocity and concentration profiles for together thermal case as displayed into Fig. 7. This has been shown that, the destructive reactions $Kr > 0$ led to falls into the concentration field this in turn deteriorates the buoyant forces impacts owing to concentration gradient. Subsequently, the flows domain is reduced. These occurrences have a superior conformity with the physically realities.

The variations of the shear stress, Nusselt numbers and Sherwood numbers are determined in Tables 1–3 for different quantities of the governing parameters. This is scrutinized from Table 1 that, it is notified that, for together ramped wall temperature and isothermal plate, the stress components τ_x as well as τ_y enhances by an

increasing in second graded fluid parameter α , chemical reacting parameter K_r , temperature generations and/or absorptions H and thermal radiation parameter R , as well as it reduces by an increasing in the permeability parameter K , thermal-diffusion (Soret) parameters S_r , thermal Grashof numbers Gr and mass Grashof quantity G_m . This is also found that by an increasing in the intensity of the magnetic fields then the stress components τ_x retards and the component τ_y boosting up for together ramped wall and isothermal plate. Table 2 illustrated the consequences of Pr , temperature generating and/or absorbing parameter H and radiating parameter R on Nusselt number. The Nusselt number Nu augment with an enhancing in Prandtl number Pr and temperature generating and/or absorbing parameter H and it retards with an increasing in radiating parameter R for together ramped wall temperature in addition to isothermal plate. Nusselt number is enhancing for ramped wall temperature in addition to it trim downs for an isothermal plate with an increasing in time. Finally, the Sherwood number Sh is reduced with an increasing in the Soret number S_r as well as Schmidt number, and it is increasing with an increasing in chemically reacting parameter K_r and certain instant of time for together ramped wall temperature and an isothermal plate (Table 3).

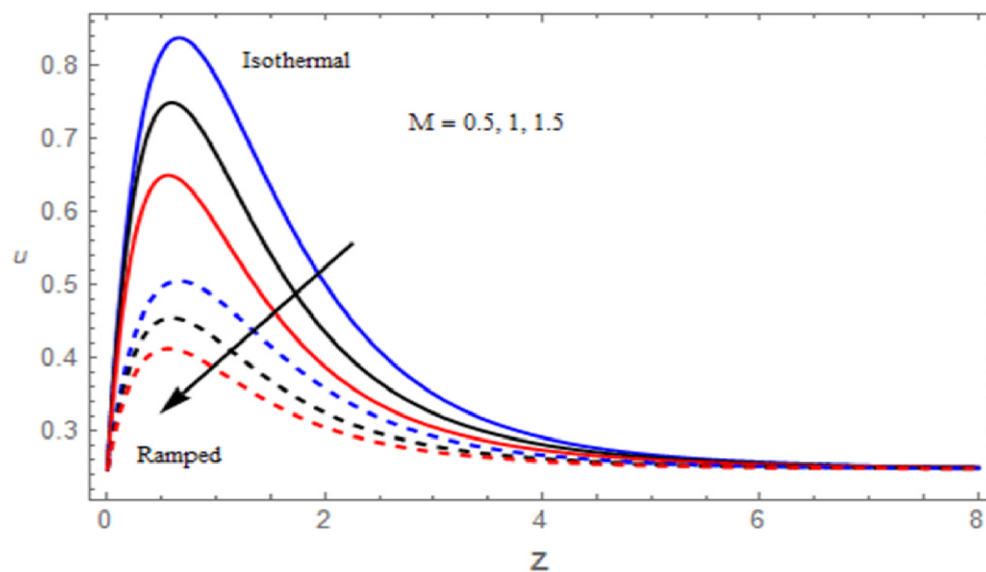


Fig.1. The Velocity profile for u against M

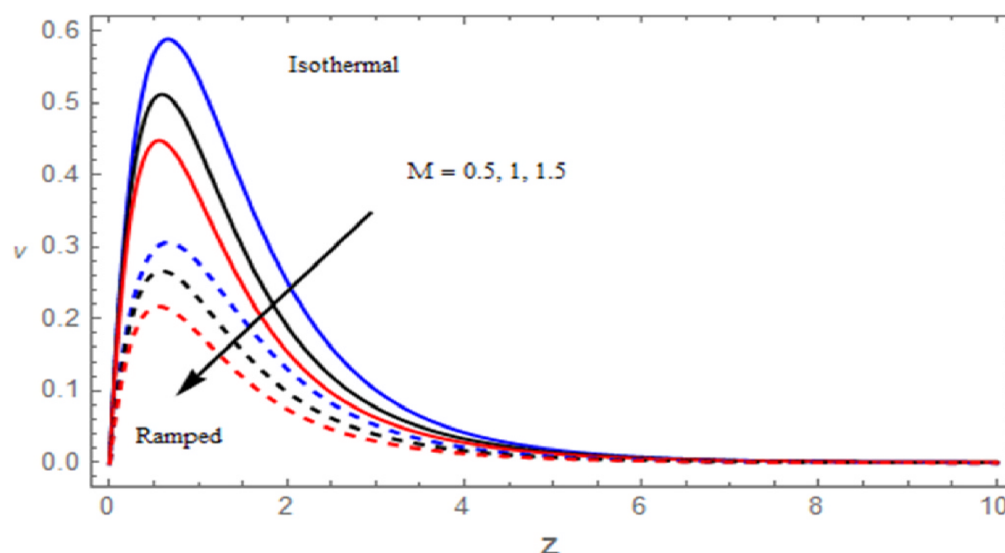


Fig.2. The Velocity profile for v against M

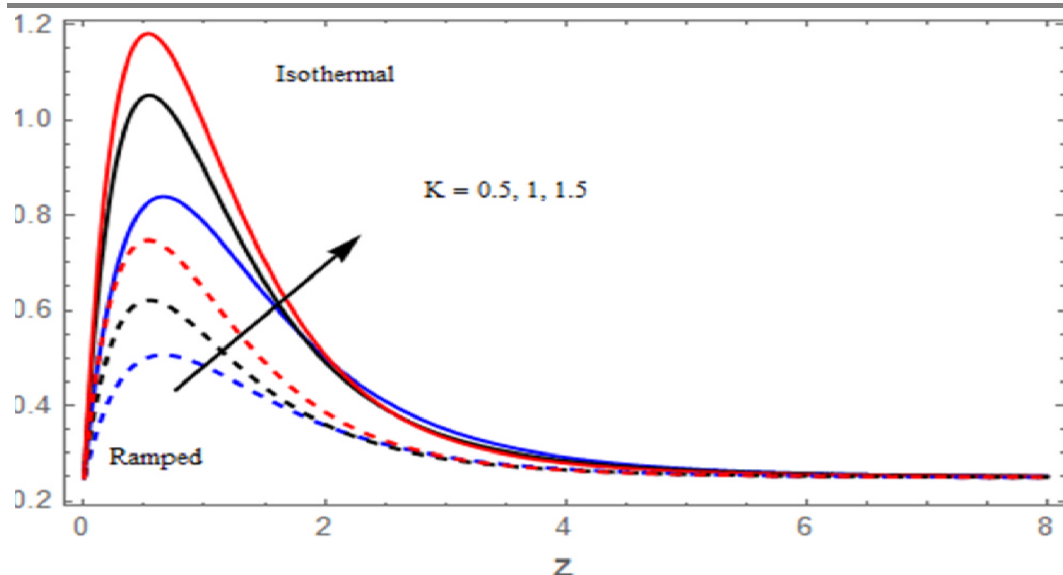


Fig.3. The Velocity profile for u against K

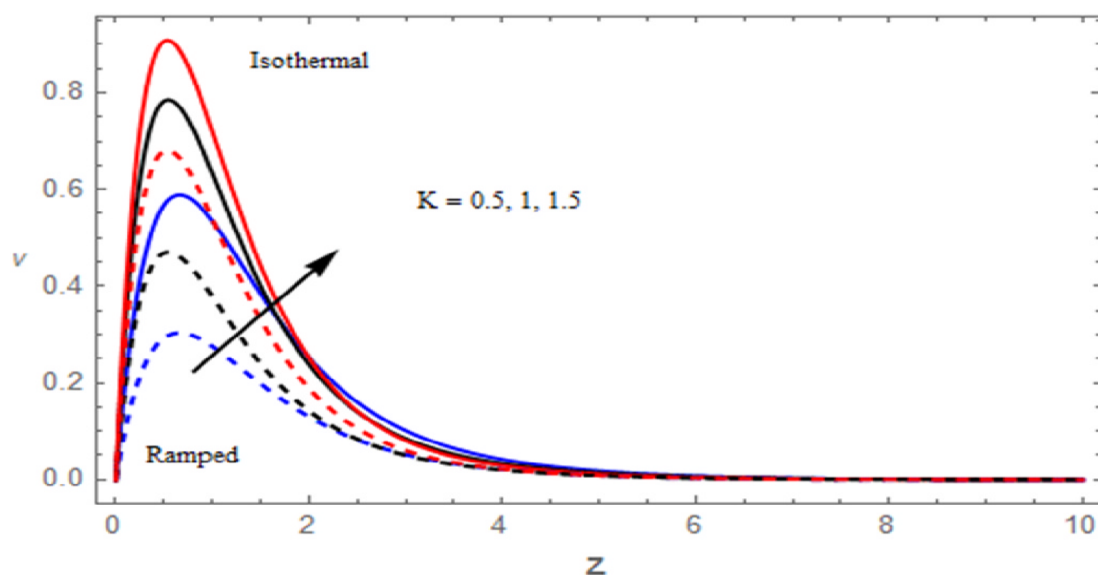


Fig.4. The Velocity profile for v against K

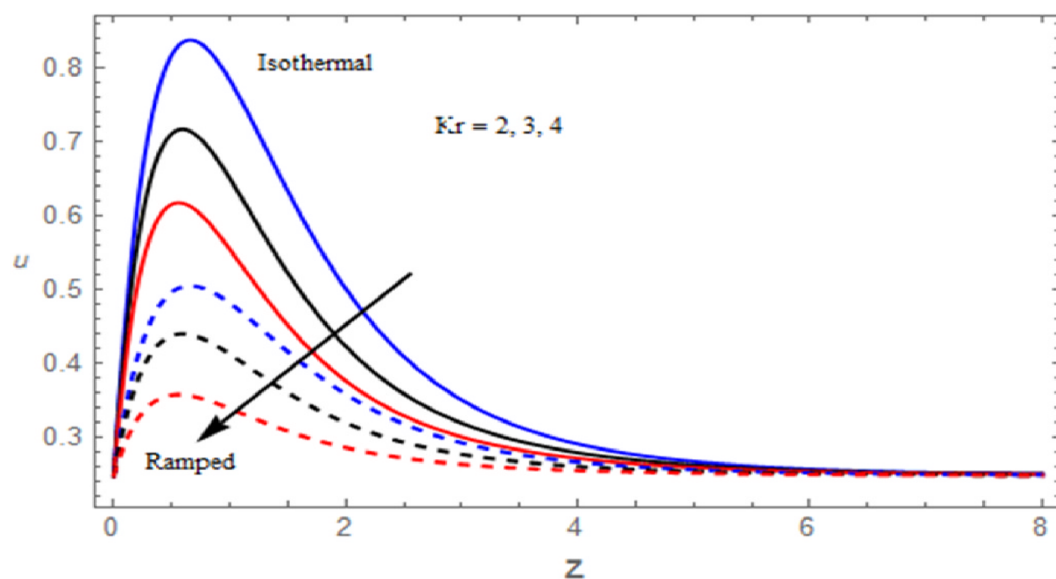


Fig.5. The Velocity profile for u against Kr

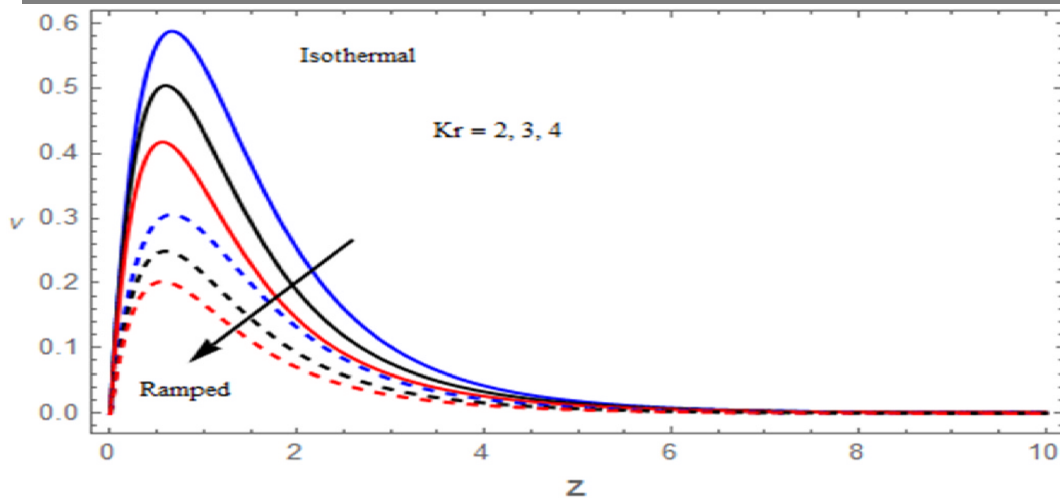


Fig.6. The Velocity profile for u against Kr

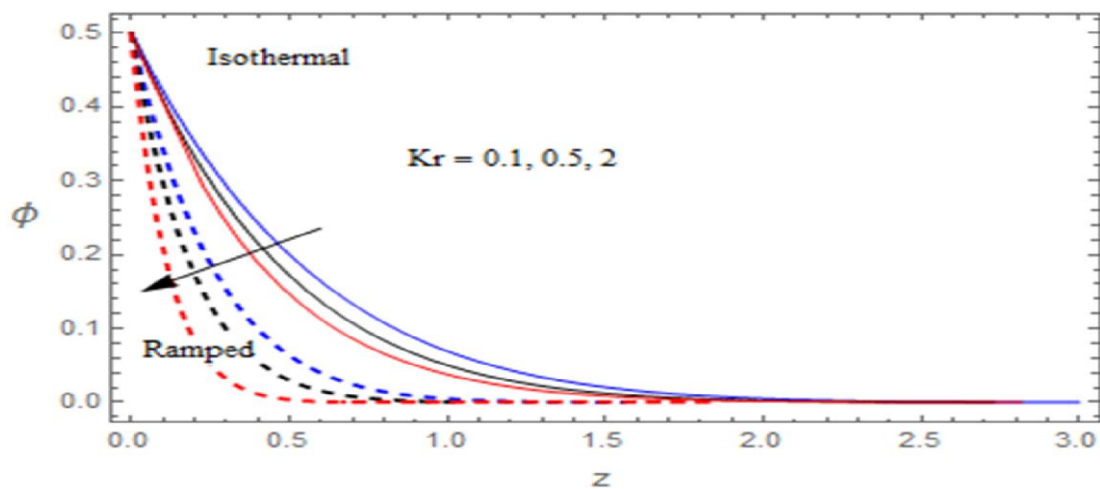


Fig.7. The Concentration profile for against Kr

Table.1. Shear stress

M	K	α	Kr	Sr	Gr	Gm	H	R	Ramped Temperature		Isothermal plate	
									τ_x	τ_y	τ_x	τ_y
0.5	0.5	0.1	2	0.1	5	2	2	2	1.238914	0.074655	1.676789	0.90178
0.8									0.819758	0.085768	1.387469	0.931437
1									0.476292	0.092448	1.157014	1.035444
	1								1.141496	0.052348	1.501635	0.835859
	1.5								0.912178	0.041372	1.356789	0.724449
		1							1.809166	0.085765	1.790966	1.034638
		1.5							2.942596	0.112417	1.932254	1.212479
			3						1.542595	0.082744	2.030801	1.101368
			4						2.175447	0.114618	3.017479	1.500931

				0.5					1.146445	0.048281	1.127884	0.582842
				1					0.956685	0.040985	0.382856	0.342548
					8				1.138812	0.051911	1.654147	0.870477
					10				0.910222	0.045744	1.632145	0.844737
						5			1.183247	0.063176	1.664265	0.893785
						8			1.076654	0.048317	1.653458	0.869904
							-5		1.146365	0.059126	1.617595	0.804638
							5		1.542356	0.085744	1.801854	1.004739
								5	1.312152	0.082159	1.790859	1.101844
								8	1.541178	0.093459	1.876855	1.501359

Table.2.Nusselt number

Pr	R	H	t	Ramped Temperature	Isothermal temperature
0.71	2	2	0.2	0.928195	1.019297
3				1.042736	1.317133
7				1.443152	1.619286
	5			0.890154	0.750059
	8			0.853935	0.467056
		-2		0.843835	0.753217
		5		1.042718	1.190074
			0.5	1.070486	0.709297
			0.8	1.068975	0.85697

Table.3.Sherwood number

Sr	Kr	Sc	t	Ramped Temperature	Isothermal temperature
0.1	2	2	0.2	0.351368	0.261569
0.5				0.268144	0.232263
1				0.644744	0.160236
	3			0.413675	0.421249
	4			0.491589	0.490095

		0.3		0.324755	0.232212
		0.6		0.291786	0.202319
			0.5	0.421759	0.502222
			0.8	0.532885	0.724107

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