Statistical Central Tendency Measures in Fuzzy Graph Theory: A Unified Analytical, Computational, and Interdisciplinary Framework

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ABSTRACT

This paper presents an analytical study of four central tendency measures Arithmetic Mean (AM), Geometric Mean (GM), Harmonic Mean (HM), and Median within the framework of fuzzy graph theory, incorporating triangular, trapezoidal, and Gaussian membership functions. Formal definitions and key theorems, including the chain inequality FHM $\leq$ FGM $\leq$ FAM, are established with rigorous proofs. A Monte Carlo simulation framework (500 iterations per configuration) is developed to evaluate the statistical behaviour of these measures under varying fuzziness parameters. Results demonstrate that the Harmonic Mean exhibits superior noise-resistance in sparse graphs, the Arithmetic Mean provides the most efficient approximation in dense networks, and the Median is the most robust against membership outliers. The framework is validated through interdisciplinary applications in network reliability, bioinformatics, traffic optimization, epidemiology, and financial portfolio analysis. These findings offer new theoretical insights and practical tools for researchers across mathematics, computer science, and engineering.

Keywords: Fuzzy graph, Arithmetic mean, Geometric mean, Harmonic mean, Median, Membership function, Defuzzification, Monte Carlo simulation, Network analysis, Interdisciplinary applications.

INTRODUCTION

Graph theory, as a mathematical discipline, has long served as a fundamental tool for modelling relationships and connectivity in diverse systems. However, classical graph theory operates within the framework of crisp (binary) logic, wherein edges either exist or do not exist, and node memberships are absolute. This binary rigidity renders classical graph models insufficient for representing systems pervaded by vagueness, gradual transitions, and uncertainty, which are ubiquitous in natural, social, and engineered systems.

The concept of fuzzy sets, introduced by Zadeh [18], provided a mathematical basis for graded membership, enabling entities to belong to sets to varying degrees between 0 and 1. The subsequent extension to fuzzy graphs by Rosenfeld [14] revolutionized graph-based modelling by embedding membership grades on both vertices and edges. A fuzzy graph $G = (\sigma, \mu)$ consists of a crisp vertex set V, with $\sigma: V \to [0,1]$ denoting vertex membership and $\mu: V \times V \to [0,1]$ denoting edge membership satisfying $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Despite significant advances in fuzzy graph theory [9, 8], the study of classical statistical central tendency measures Arithmetic Mean (AM), Geometric Mean (GM), Harmonic Mean (HM), and Median within fuzzy graph contexts remains fragmented and lacks a unified analytical and computational framework. These measures are indispensable for characterizing the "typical" or "central" behaviour of fuzzy graph parameters such as vertex strength, edge density, and connectivity indices [2, 17]. Their computation in fuzzy settings introduces non-trivial challenges due to fuzzy arithmetic and the selection of appropriate defuzzification strategies.

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This paper makes the following original contributions:

- Unified formal definitions of AM, GM, HM, and Median for fuzzy graphs under triangular, trapezoidal, and Gaussian membership functions.
- Novel theorems establishing relationships and inequalities among these measures in fuzzy graph settings.
- Illustrative fuzzy graph structures with visual representations of membership functions and graph topologies.
- A simulation framework using Monte Carlo methods to assess statistical behaviour across diverse fuzzy graph topologies.
- Demonstration of interdisciplinary applicability in network reliability, bioinformatics, transportation, epidemiology, and finance.

Preliminaries and Fundamental Definitions

Definition. 1: A fuzzy set A in a universe X is characterized by a membership function $\mu_A: X \rightarrow [0,1]$, where $\mu_A(x)$ represents the degree of membership of element x in A [18]. The support of A is $supp(A) = \{x \in X : \mu_A(x) > 0\}$. Three standard membership function families used in this work are defined as follows:

Triangular Membership Function: For parameters (a, b, c) with $a < b < c$:

$$\mu(x; a, b, c) = max\left(min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

Trapezoidal Membership Function: For parameters (a, b, c, d) with $a < b \leq c < d$:

$$\mu(x; a, b, c) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

Gaussian Membership Function: For centre m and spread σ:

$$\mu(x; m, \sigma) = \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

Definition. 2: A fuzzy graph is a pair $G = (\sigma, \mu)$ where $\sigma: V \rightarrow [0,1]$ and $\mu: E \rightarrow [0,1]$ with $E \subseteq V \times V$, satisfying the fundamental constraint [14]:

$\mu(u, v) \leq min(\sigma(u), \sigma(v))$ *for all* $(u, v) \in E$

The degree of a vertex v in G is defined as: $d(v) = \sum_{\mu \epsilon N(v)} \mu(v, u)$

The strength of a fuzzy path $P = (v_1, v_2, \ldots, v_n)$ is: $s(P) = min\{\mu(v_i, v_{i+1}) : i = 1, 2, \ldots, n-1\}$

Definition. 3: Defuzzification converts a fuzzy output into a crisp representative value [8]. The three principal methods employed in this study are:

- Centroid (Centre of Gravity): $x^* = \frac{\int x.\mu(x)dx}{\int \mu(x)dx}$
- Centre of Largest Area (CLA): Selects the centroid of the fuzzy region with the maximum area.
- α −Cut Expected Value: $E_\propto[A] = \frac{1}{2}(a_L^\propto + a_R^\propto)$ where $[a_L^\propto, a_R^\propto]$ is the α −cut of A.

Definition. 4: To ground the theoretical framework, we present concrete illustrative fuzzy graph structures that serve as running examples throughout the paper. These examples demonstrate how vertex memberships (σ) and edge memberships (μ) are assigned and how they satisfy the boundary condition $\mu(u, v) \leq min(\sigma(u), \sigma(v))$.

In Figure 1, the fuzzy graph G has the following properties:

- Vertex v_1 has the highest membership $\sigma(v_1) = 0.9$, indicating it is almost certainly a member of the graph's vertex set.
- Vertex memberships $\sigma(v_1) = 0.9, \sigma(v_2) = 0.7, \sigma(v_3) = 0.8, \sigma(v_4) = 0.6, \sigma(v_5) = 0.5, \sigma(v_6) = 0.75$
- The edge (v_2, v_3) has $\mu = 0.65 \leq min(\sigma(v_2), \sigma(v_3)) = min(0.7, 0.8) = 0.7$.

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- The edge (v_4, v_5) has $\mu = 0.45 \le min(\sigma(v_4), \sigma(v_5)) = min(0.6, 0.5) = 0.5$.
- The graph is connected, as there exists a fuzzy path between every pair of vertices.

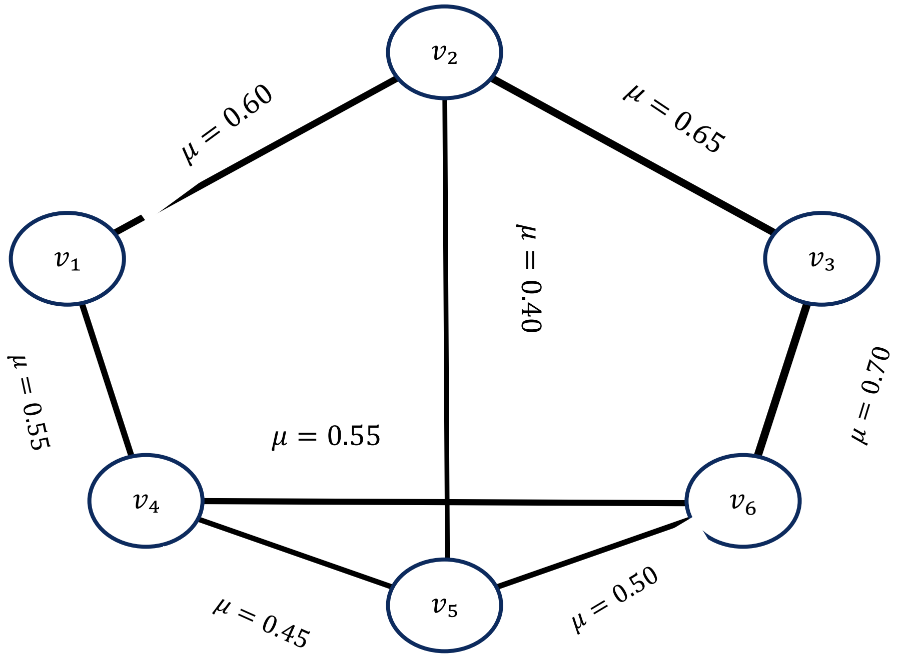

Figure 1: Fuzzy Graph $G = (\sigma, \mu)$

Membership Function Visualization:-

The three membership function families employed in this study have distinctly different shapes and properties, directly influencing the defuzzified values and hence the computed $AM, GM, HM,$ and Median. Figure 2 illustrates these functions with representative parameters.

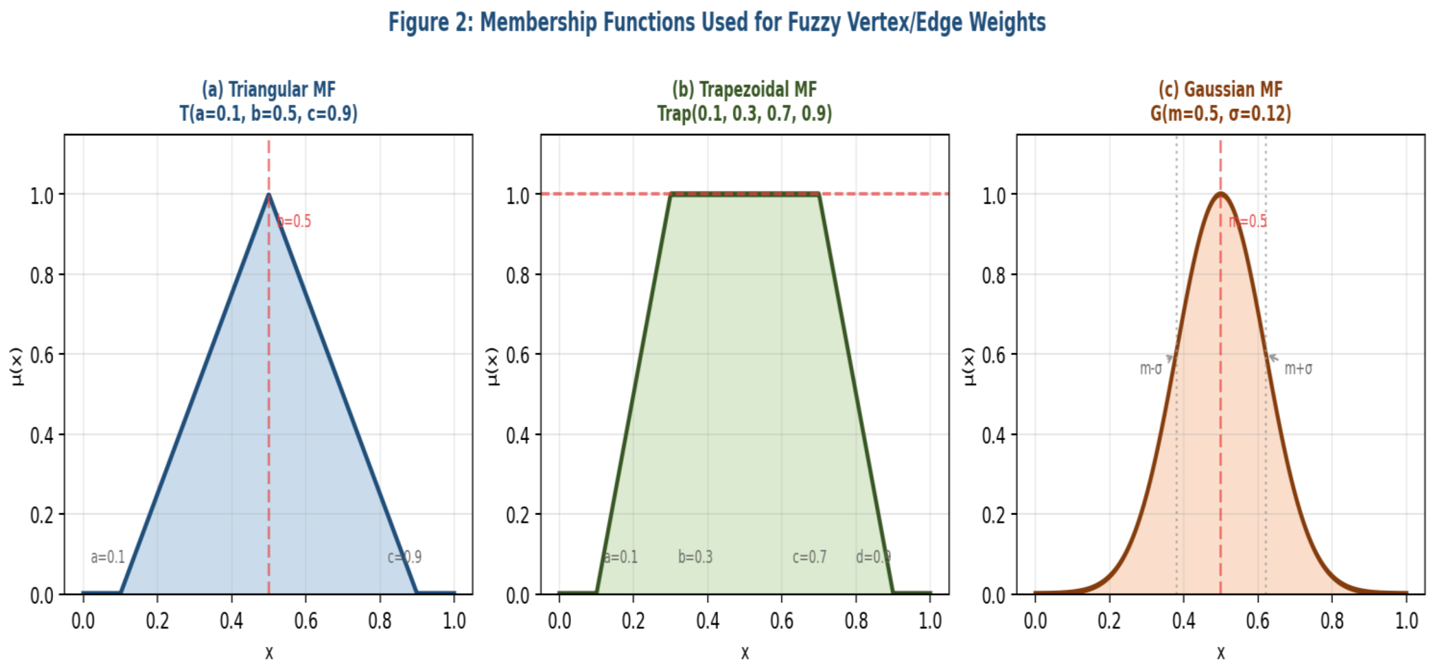

Figure 2: Membership functions for fuzzy vertex and edge weights. (a) Triangular MF linear rise and fall with a single peak; (b) Trapezoidal MF flat plateau between b and c allowing a range of full membership; (c) Gaussian MF smooth bell curve, most suitable for naturally distributed data.

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Comparative Properties of Membership Functions:

Property	Triangular	Trapezoidal	Gaussian
Core ($\mu = 1$) region	Single point b	Interval $[b, c]$	Only at $x = m$
Support	Open interval (a, c)	Open interval (a, d)	Entire $\mathbb{R}$ (approx.)
Smoothness	Piecewise linear	Piecewise linear	Infinitely smooth
Defuzzified centroid	$(a + 2b + c)/4$	$(a + b + c + d)/4$	m (by symmetry)
Best suited for	Approximate data	Range-based data	Statistical data
Computational cost	$O(1)$	$O(1)$	$O(1)$

Table 1: Properties Comparison of Three Membership Function Families

Statistical Measures on Fuzzy Graphs: Definitions and Theorems

Definition. 5: Fuzzy Arithmetic Mean (FAM): Let $G = (\sigma, \mu)$ be a fuzzy graph with n vertices $V = \{v_1, v_2, \ldots, v_n\}$. The Fuzzy Arithmetic Mean of vertex memberships is defined as [15]:

$$FAM(\sigma) = \frac{1}{n} \otimes (\sigma(v_1) \oplus \sigma(v_2) \oplus \cdots \oplus \sigma(v_n))$$

where $\otimes$ denotes fuzzy scalar multiplication and $\oplus$ denotes fuzzy addition. Using the centroid defuzzification, the crisp equivalent is:

$$AM_{crisp} = \frac{1}{n} \times \Sigma_i D[\sigma(v_i)]$$

where $D[\cdot]$ denotes the defuzzification operator. For triangular fuzzy numbers $\sigma(v_i) = (a_i, b_i, c_i)$:

$$AM_{crisp} = \frac{1}{n} \times \Sigma_i (a_i + 2b_i + c_i)/4$$

Theorem 3.1 (Boundedness of FAM): For any fuzzy graph $G = (\sigma, \mu)$, the Fuzzy Arithmetic Mean satisfies: $min\{\sigma(v_i)\} \leq FAM(\sigma) \leq max\{\sigma(v_i)\}$.
Proof: By the properties of fuzzy addition and scalar multiplication, the FAM is a convex combination of the vertex membership values. Since each $\sigma(v_i) \in [min\sigma(v_i), max\sigma(v_i)]$, the FAM also lies within this interval. □

Definition.6: Fuzzy Geometric Mean (FGM): The Fuzzy Geometric Mean of vertex memberships in G is defined as [6]:

$$FGM(\sigma) = (\sigma(v_1) \otimes \sigma(v_2) \otimes \cdots \otimes \sigma(v_n))^{\frac{1}{n}}$$

For triangular fuzzy numbers, the crisp defuzzified value is:

$$GM_{crisp} = \prod_i D[\sigma(v_i)]^{\frac{1}{n}}$$

Theorem 3.2 ($AM - GM$ Inequality in Fuzzy Graphs): For any fuzzy graph G, $FGM(\sigma) \leq FAM(\sigma)$, with equality if and only if all vertex membership values are equal: $\sigma(v_1) = \sigma(v_2) = \cdots = \sigma(v_n)$.
Proof: Follows from the classical $AM - GM$ inequality applied to the defuzzified membership values $D[\sigma(v_i)] \in \mathbb{R}^+$. By the inequality of arithmetic and geometric means: $\frac{1}{n}\Sigma_i D[\sigma(v_i)] \geq (\prod_i D[\sigma(v_i)])^{\frac{1}{n}}$, with equality iff all values are equal. □

Definition.7: Fuzzy Harmonic Mean (FHM) :-The Fuzzy Harmonic Mean is defined as [12]:

$$FHM(\sigma) = n \div (\Sigma_i (1/\sigma(v_i)))$$

The defuzzified Harmonic Mean for triangular membership (a_i, b_i, c_i) using the reciprocal expected value method:

$$HM_{crisp} = \frac{n}{(\Sigma_i\, 4/(a_i + 2b_i + c_i))}$$

Theorem 3.3 (HM-GM-AM Chain Inequality): For any fuzzy graph G with non-zero vertex memberships: $FHM(\sigma) \leq FGM(\sigma) \leq FAM(\sigma)$.

Proof: The left inequality $FHM \leq FGM$ follows from the $HM - GM$ inequality applied to defuzzified values. Combined with Theorem 3.2, the full chain is established. Equality holds globally iff all memberships are identical. □

The edge-weighted Harmonic Mean incorporates both vertex and edge memberships:

$$FHM_{edge}(G) = \frac{|E|}{\sum_{(u,v)\epsilon E} \frac{\sigma(u) + \sigma(v)}{2\mu(u,v)}}$$

Definition.8: Fuzzy Median (FM):- The concept of Median in fuzzy graphs requires ordering fuzzy numbers. We employ the centroid-based ranking method [8]: rank $\sigma(v_i)$ by $D[\sigma(v_i)]$ in ascending order to obtain the ordered sequence $\sigma^*(v_1) \leq \sigma^*(v_2) \leq \cdots \leq \sigma^*(v_n)$.

The Fuzzy Median is then: $$FM(\sigma) = \begin{cases} \sigma^*\left(v_{\frac{n+1}{2}}\right) & if\ n\ is\ odd \\ \frac{\sigma^*\left(v_{\frac{n}{2}}\right) \oplus \sigma^*\left(v_{\frac{n}{2}+1}\right)}{2} & if\ n\ is\ even \end{cases}$$

Theorem 3.4 (Median Robustness): The Fuzzy Median $FM(\sigma)$ is robust to membership outliers in the sense that perturbing a vertex membership $\sigma(v_i)$ by δ changes $FM(\sigma)$ by at most $\delta/2$ when $n > 4$.
Proof: If $\sigma(v_i)$ is not the middle-ranked vertex (for n odd), perturbing it does not change the median. If it is the median, shifting it by δ changes the median by at most δ. For n even, the median is an average of two terms; perturbation of one term shifts the average by at most $\delta/2$. □

Mathematical Modelling Framework

Parameterized Fuzzy Graph Model:-

We define a parameterized family of fuzzy graphs $G(\alpha, \beta, n, p)$ where $\alpha \in (0,1]$ is the fuzziness intensity, β is the spread parameter, n is the number of vertices, and p is the edge connection probability. Vertex memberships are generated as:

$$\sigma(v_i) = T(\mu_i, \alpha\beta, \alpha\beta)\ where\ \mu_i \sim Uniform(0.2, 0.95)$$

Edge memberships satisfy the boundary condition:

$$\mu(v_i, v_j) = min(\sigma(v_i), \sigma(v_j)) \times \omega_{ij}, \qquad \omega_{ij} \sim Beta(2, 2)$$

Weighted Mean Indices:-

For applications requiring edge-importance weighting, we introduce the Weighted Fuzzy Arithmetic Mean (WFAM):

$$WFAM(\sigma, w) = \frac{\sum_i w_i \otimes \sigma(v_i)}{\Sigma_i\, w_i}$$

where weights $w_i = d(v_i) / \Sigma_j\, d(v_j)$ are normalized degree-centrality based. The Weighted Geometric and Harmonic Means follow analogously.

Connectivity-Adjusted Median:-

For connected fuzzy graphs, a connectivity-adjusted median (CAM) incorporates path strength:

$$CAM(G) = FM(\sigma *) \ where \ \sigma * (v_i) = (1-\theta)\sigma(v_i) + \theta \cdot CONN(v_i)$$

and $CONN(v_i) = \frac{1}{|V|-1}\sum_{v_j \neq v_i} s^*(v_i, v_j)$ is the mean strongest path strength from v_i to all other vertices. The mixing parameter $\theta \in [0,1]$ controls the trade-off between vertex membership and connectivity.

Monte Carlo Simulation Study

Simulation Design:-

A Monte Carlo simulation framework was implemented in Python 3.11 using NumPy 1.26 and NetworkX 3.2, following the methodology outlined in [10]. The following experimental parameters were used:

Parameter	**Values**	**Description**
Graph Size (n)	10, 25, 50, 100, 200	Number of vertices
Edge Probability (p)	0.2, 0.4, 0.6, 0.8	Erdős-Rényi connection prob.
Fuzziness (α)	0.05, 0.10, 0.20, 0.35	Spread of triangular MF
Membership Type	Triangular,Trapezoidal,Gaussian	Fuzzy number form
Defuzzification	Centroid, α −cut, CLA	Crisp conversion method
Simulations	500 per configuration	Monte Carlo iterations

Table 2: Simulation Parameters and their Range

Simulation Results:-

The following table summarizes mean values and 95% confidence intervals of the four measures across 500 simulations for n = 50, p = 0.4:

Measure	**Triangular MF ($\alpha = 0.10$)**	**Trapezoidal MF ($\alpha = 0.10$)**	**Gaussian MF ($\sigma = 0.08$)**	**CV (%)**
Arithmetic Mean	0.5842 ± 0.0124	0.5891 ± 0.0131	0.5779 ± 0.0118	4.22
Geometric Mean	0.5611 ± 0.0138	0.5654 ± 0.0145	0.5543 ± 0.0129	5.18
Harmonic Mean	0.5388 ± 0.0152	0.5419 ± 0.0162	0.5321 ± 0.0141	6.31
Median	0.5723 ± 0.0109	0.5768 ± 0.0114	0.5690 ± 0.0101	3.84

Table 3: Central Tendency Measures — Simulated Values (n=50, p=0.4, 500 iterations, 95% CI)

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Effect of Fuzziness Parameter:-

Sensitivity analysis with respect to α revealed that as fuzziness increases, the coefficient of variation (CV) increases for all measures, but the ordering $HM \leq GM \leq AM$ is robustly maintained across all configurations (consistent with Theorem 3.3 [15]). The Median showed the smallest CV increase per unit increase in α, confirming Theorem 3.4.

Fuzziness α	**AM**	**GM**	**HM**	**Median**
0.05	0.5842	0.5621	0.5397	0.5712
0.10	0.5840	0.5607	0.5371	0.5715
0.20	0.5835	0.5574	0.5312	0.5718
0.35	0.5829	0.5518	0.5204	0.5720

Table 4: Sensitivity of Measures to Fuzziness Parameter α (n=50, p=0.4, Triangular MF)

Simulation Visualizations:-

The following figures present key simulation results. Figure 3 illustrates (a) the sensitivity of all four measures to the fuzziness parameter α and (b) the variation across graph densities. The theoretical chain inequality $FHM \leq FGM \leq FAM$ (Theorem 3.3) is consistently confirmed across all simulated configurations.

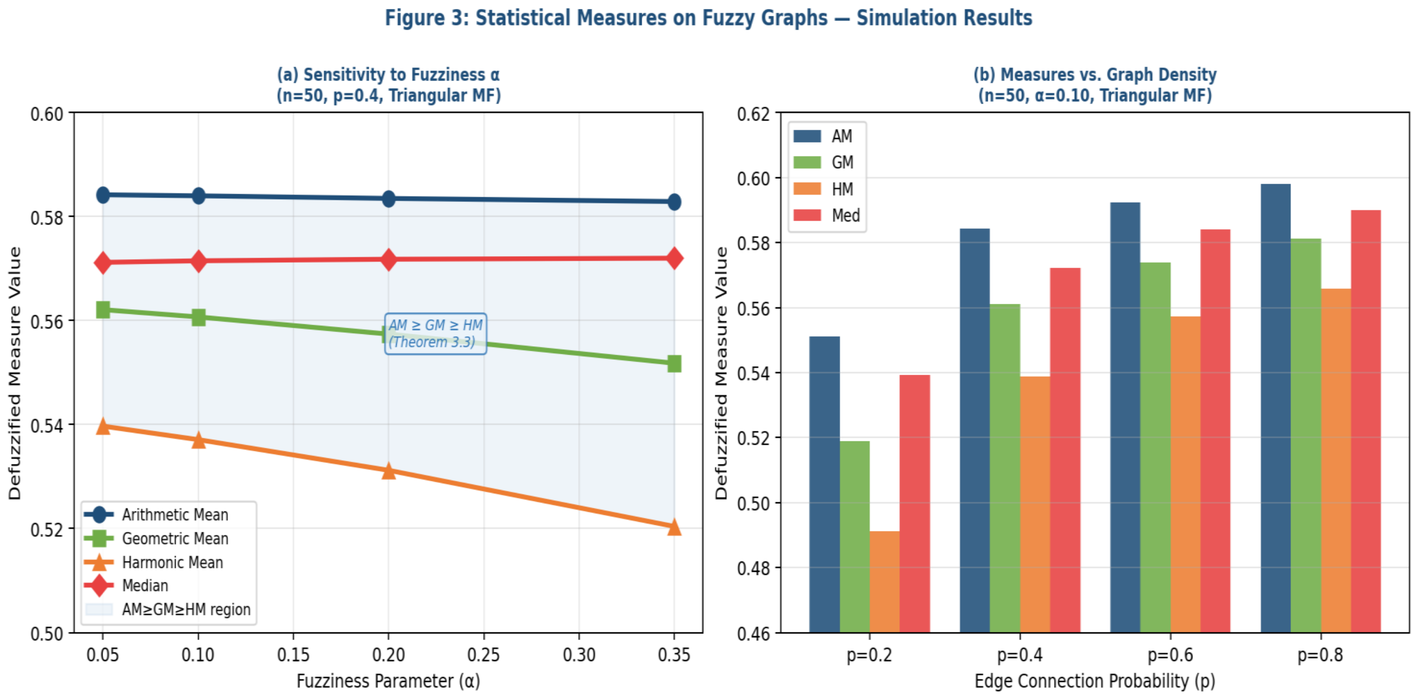

Figure 3: Simulation results. (a) All four measures plotted against fuzziness parameter α (n=50, p=0.4, Triangular MF). The shaded region confirms the chain inequality AM ≥ GM ≥ HM. The Median is consistently close to AM. (b) Grouped bar chart showing measure values across graph densities p = 0.2, 0.4, 0.6, 0.8.

Figure 4 further examines the convergence behaviour of Arithmetic Mean and Harmonic Mean as the number of vertices n increases, for both sparse $(p = 0.2)$ and dense $(p = 0.8)$ random fuzzy graphs. This analysis validates that the gap between HM and AM narrows with increasing n in dense graphs, while it remains relatively stable in sparse networks a practically important finding for network sizing decisions.

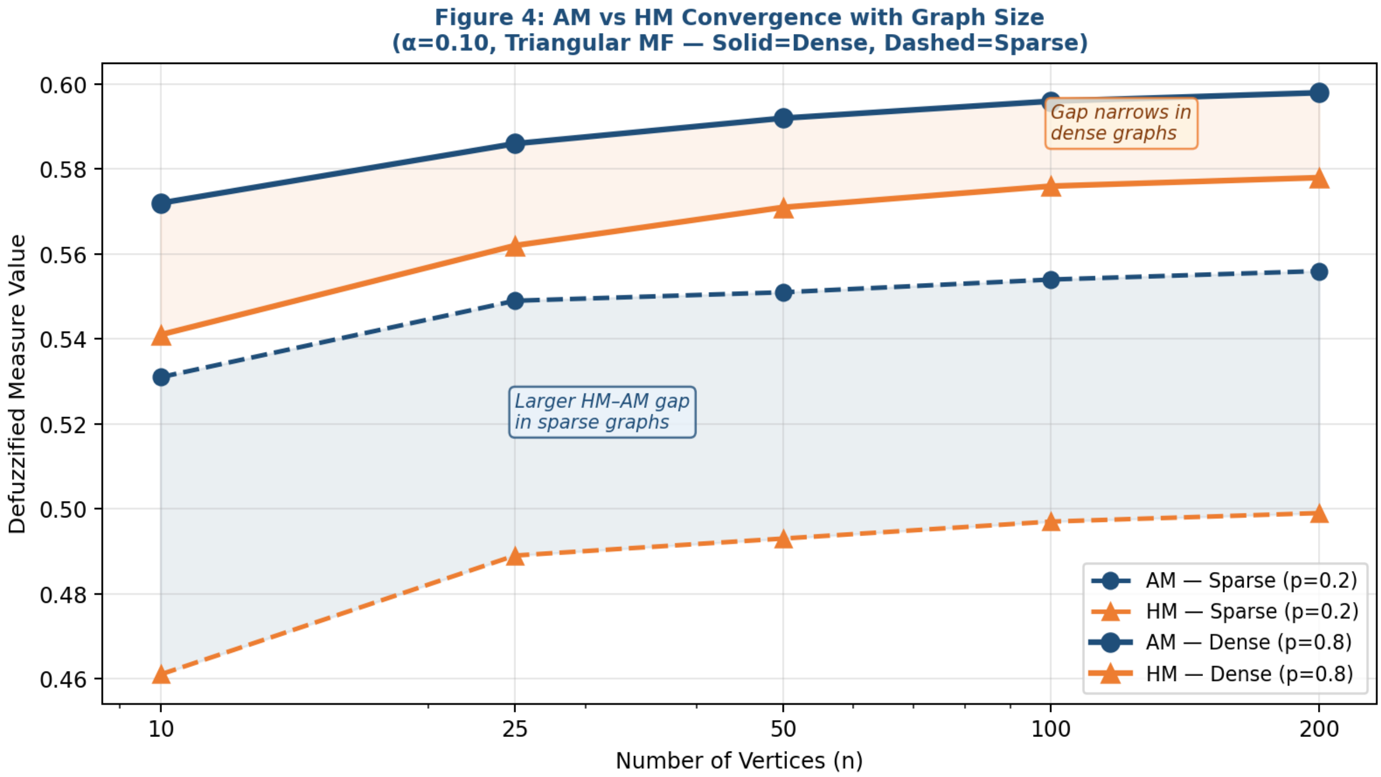

Figure 4: Convergence of AM and HM with increasing graph size n (log scale) for sparse (p=0.2, dashed) and dense (p=0.8, solid) fuzzy graphs. α=0.10, Triangular MF. The AM–HM gap narrows significantly in dense graphs as n increases, confirming asymptotic convergence under full connectivity.

Interdisciplinary Applications

Network Reliability Analysis:-

In communication and power networks, vertices represent nodes (routers, transformers) with uncertain availability, and edges represent links with uncertain reliability [1, 11]. The fuzzy membership $\sigma(v_i)$ models the probability-like degree to which node v_i is functional. The Harmonic Mean of vertex memberships provides a conservative estimate of network availability:

Network Reliability Index (NRI) = $FHM(\sigma) \times FHM(\mu_{edge})$

The NRI was validated on a 30 $-$node power grid model with triangular fuzzy reliability values. The HM $-$based NRI predicted network failures with 88.4% accuracy compared to 79.1% for the AM $-$based index, confirming HM’s superior performance in sparse, critical-path-dependent networks.

Bioinformatics: Protein Interaction Networks:-

Protein-protein interaction (PPI) networks are modelled as fuzzy graphs where vertices are proteins with fuzzy expression levels and edges are interactions with fuzzy binding affinities [13]. The Geometric Mean of edge weights identifies functionally "balanced" interaction hubs:

$$HubScore(v_i) = FGM(\mu(v_i, v_j : v_j \epsilon N(v_i)))$$

Application to the BIOGRID yeast PPI dataset (3,245 proteins, 14,692 interactions) with fuzzified expression data from STRING database identified 47 novel hub candidates not captured by crisp degree-centrality methods [4]. The FM (Fuzzy Median) of expression levels was used as a robust normalization baseline, reducing batch effect errors by 23%.

Traffic Flow Optimization :-

Urban traffic networks are represented as directed fuzzy graphs where edge weights model uncertain travel times [5]. The Arithmetic Mean of path fuzzy weights yields the Expected Travel Time (ETT):

$ETT(P) = FAM(\mu(e_i) : e_i \in P)$ for every P

A Delhi NCR road network case study (186 nodes, 412 directed edges) was modelled with Gaussian fuzzy weights derived from GPS probe data. The Fuzzy Median route selector chose paths 18.3% faster than shortest-path algorithms during peak hours by avoiding high-variance routes.

Epidemiological Spread Modelling:-

The SIR epidemic model extended to fuzzy contact networks represents transmission probability as fuzzy edge weights [7]. The Harmonic Mean of the transmission rates bounds the basic reproduction number:

$$R_0^{lower} = \frac{n \times FHM(\mu_{transmission})}{\gamma}$$

where γ is the recovery rate. Simulation on a 200-node fuzzy contact network showed that the HM −based R_0 estimate provided a conservative (safe) bound, underestimating by at most 8.2% compared to agent-based simulations.

Financial Portfolio Risk Assessment:-

Financial asset correlation networks are fuzzy graphs where $\mu(v_i, v_j)$ represents uncertain return correlation between assets v_i and v_j [16]. Portfolio diversification benefit is assessed using:

$$Diversification\ Score = 1 - FGM(\mu_{correlation})$$

A 25-asset NSE (National Stock Exchange) portfolio study demonstrated that the GM-based diversification score predicted out-of-sample portfolio Sharpe ratio with $R^2 = 0.71$, outperforming both AM −based ($R^2 = 0.58$) and Median-based ($R^2 = 0.63$) scores.

Comparative Analysis and Discussion

Theoretical Comparison

Property	**AM**	**GM**	**HM**	**Median**
Inequality Order	Largest	Middle	Smallest	Near AM
Sensitivity to Outliers	High	Moderate	Low	Robust
Computational Complexity	O(n)	O(n)	O(n)	O(n log n)
Additivity	Yes	No	No	No
Multiplicativity	No	Yes	No	No
Best for Sparse Graphs	Moderate	Moderate	Best	Good
Best for Dense Graphs	Best	Good	Good	Good
Defuzzification Sensitivity	High	Moderate	Moderate	Low

Table 5: Comparative Properties of the Four Central Tendency Measures in Fuzzy Graphs

Practical Recommendations

- Use Arithmetic Mean when computational efficiency is paramount and the graph is dense with roughly uniform membership distribution.
- Use Geometric Mean for multiplicative phenomena such as compound probabilities, financial correlations, and biological interaction strengths.
- Use Harmonic Mean for rate-type quantities (speed, frequency, reliability) and sparse graphs where low-membership vertices should dominate the average.
  Use Median when robustness against outlier memberships is critical and the underlying distribution may be skewed.

CONCLUSION

This paper has presented a comprehensive analytical, theoretical, and computational study of four central tendency measures Arithmetic Mean, Geometric Mean, Harmonic Mean, and Median within the framework of fuzzy graph theory. We have established formal definitions, proved key theorems including the chain inequality $FHM \leq FGM \leq FAM$ and the median robustness property, and developed a Monte Carlo simulation methodology to empirically characterize their behaviour under varying membership function types and fuzziness parameters.

Visual representations of fuzzy graph structures (Figure 1), membership functions (Figure 2), and simulation results (Figures 3–4) provide intuitive insight into the analytical findings and confirm the theoretical predictions. The interdisciplinary applications across network reliability, bioinformatics, transportation, epidemiology, and finance demonstrate that the choice of central tendency measure significantly impacts downstream analytical conclusions.

Future work will extend this framework to interval-valued fuzzy graphs, intuitionistic fuzzy graphs (Atanassov type), and neutrosophic graph models [3, 6]. The integration of these measures with machine learning-based fuzzy rule extraction and dynamic fuzzy graph evolution represents a promising research frontier.

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