

Exploring Vedic Sutra Techniques for Solving Ordinary Differential Equations

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ABSTRACT

The primary aim of this paper is to examine the use of Vedic Mathematics sutras in deriving derivatives and solving ordinary differential equations. Vedic sutras such as Urdhva-Tiryagbhyam, Ekanyunena Purvena, Calana-Kalanabhyam, Gunakasamuccayah, and Lopanasthapanabhyam helps to simplify the computational process. The study seeks to establish the relevance and efficiency of these Vedic principles in differentiation and differential equation solving. Employing Vedic methods for obtaining derivatives and solutions significantly reduces computational complexity and enhances mental calculation speed. Some illustrative examples are presented to demonstrate the application of these sutras in differentiation. The findings indicate that Vedic Mathematics offers a simplified, faster, and more intuitive approach to solving ordinary differential equations.

Keywords: Vedic Mathematics, Differential Equations, Derivatives, Urdhva-Tiryagbhyam Sutra, Calana-Kalanabhyam Sutra, Ekanyunena Purvena, Gunakasamuccayah, Lopanasthapanabhyam.

INTRODUCTION

Vedic Mathematics is a collection of sutras and sub-sutras introduced by Maharaja [10], forming a mathematical system rooted in the ancient Vedas. These sixteen sutras, along with their sub-sutras, can be applied across a wide range of mathematical disciplines, including coordinate geometry, linear equations, number theory as well as differential and integral calculus [15]. In this paper, we focus on the application of these sutras to solve ordinary differential equations. Several researchers have contributed to the understanding and application of Vedic sutras in advanced calculus. Kumar et al. [5] and Kumari et al. [6] demonstrated the use of these sutras in differential and integral calculus, providing brief explanations of the sutras and applying them to various types of equations to compute derivatives and integrals. Devkumar [2] and Mandloi [9] provided a comprehensive overview of Vedic sutras, emphasizing their logical framework and demonstrating that Vedic Mathematics offers greater efficiency and integration compared to conventional mathematical methods. Several researchers have also investigated the historical origins of mathematical ideas within Vedic literature. Garrain et al. [3] analyzed the contributions of post-Vedic mathematicians and concluded that the development of calculus was more advanced in India than in many Western countries. Gupta [4] conducted an extensive study of all the sutras, applying them to diverse mathematical branches, and found that Vedic techniques can effectively address a wide range of problems. Similarly, Vyas [14] discussed the historical importance, practical value, and wide applicability of Vedic Mathematics in his conceptual analysis. Prasad [12] and Kumari [7] presented illustrative summaries of essential sutras, demonstrating that their application in basic computations significantly reduces time and simplifies operations. Lal et al. [8] concentrated on the sutras “Ekadhikena” and “Ekanyunena,” elucidating their use in determining derivatives and antiderivatives of functions. Singh et al. [13] underscored the role of Vedic Mathematics in fostering intellectual and mental growth among learners. Their study applied various sutras and sub-sutras to solve homogeneous differential equations with constant coefficients, revealing that the Vedic approach streamlines the solving process and enhances practicality. Extending this line of research, Priya et al. [11] utilized Vedic sutras to solve differential equations and integrals involving partial fractions, noting improved computational speed and accuracy in advanced mathematics. Finally, Anjali et al. [1] explored the connection between Vedic sutras and higher calculus, emphasizing their practical significance in modern mathematical problem-solving.

In the present study, we extend this approach by applying Vedic sutras to different ordinary differential equations, supported by illustrative examples. The structure of this paper includes objectives in section 2. a brief explanation of relevant sutras in Section 3, their application in Section 4, followed by examples in Section 5. The study concludes that Vedic sutras provide an effective method for solving various types of differential equations.

Objectives

The objectives of this study are:

- To demonstrate the application of selected Vedic Sutras in solving ordinary differential equations.
- To simplify differentiation processes using Vedic principles.
- To analyze computational efficiency and learning advantages of these techniques.
- To present examples illustrating the effectiveness of Vedic methods compared to conventional calculus.

Applied Sutras

The following Vedic sutras have been employed in this study, each contributing a unique approach to solving problems in calculus and differential equations.

Calana-Kalanabhyam Sutra

The Calana-Kalanabhyam Sutra, meaning “*by calculus*” or “*by the principle of change and computation*”, plays an important role in the study of ordinary differential equations (ODEs). This sutra emphasizes the concept of rate of change, which is the fundamental idea behind differentiation.

In the context of ordinary differential equations, Calana-Kalanabhyam Sutra is used to identify and analyze the first differential of a function directly from its algebraic form. It provides an intuitive approach to determine how a dependent variable changes with respect to an independent variable, without relying on lengthy classical procedures.

For polynomial and quadratic functions, this sutra establishes a relationship between the algebraic properties of the equation (such as coefficients or discriminant) and the behaviour of its derivative. Hence, it helps in simplifying the formation and solution of differential equations by linking algebraic structure with calculus-based variation.

Ekanyunena Purvena Sutra

The *Ekanyunena Purvena* Sutra, a sub-sutra of *Nikhilam*, literally means “By one less than the previous one” In differential calculus, it is applied to find the derivative of a power function x^n , where n is any real number. According to this sutra When you differentiate a polynomial the power of x decreases by 1 each time — exactly matching the idea of “By one less than the previous one.” and by multiplying the term by its power and then reducing the index by one, as expressed below:

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

This provides a simple and direct approach to differentiation of power functions.

Gunakasamuccayah Sutra

Gunakasamuccayah Sutra states that “*the product of the sum of the coefficients in the factors is equal to the sum of the coefficients of the product.*” In algebra, this sutra is widely used for quick factorization and verification of polynomial expressions. In the context of **ordinary differential equations (ODEs)**, this sutra helps in **simplifying auxiliary (characteristic) equations**, verifying solutions, and checking the consistency of differential operators.

Conceptual Role in ODEs

In linear ordinary differential equations with constant coefficients, the differential operator can be treated algebraically. The Gunakasamuccayah Sutra is applied to:

- verify factorized forms of differential operators,
- simplify auxiliary equations,
- ensure correctness of solutions obtained through factorization.

Urdhva-Tiriyagbhyam Sutra

The Urdhva-Tiriyagbhyam Sutra, which translates as “vertically and crosswise,” It is the general formula for multiplication of any two numbers or algebraic expressions.

But its deeper meaning thinking both vertically (direct) and crosswise (indirect) connects beautifully to the concept of relationships between variables in calculus.

In arithmetic or algebra, Urdhva–Tiryagbhyām is used to multiply numbers or expressions using the vertical and crosswise method:

$$(a + b)(c + d) = ac + (ad + bc) + bd$$

Here,

- Vertical terms (ac, bd) represent direct relationships,
- Crosswise terms (ad + bc) represent interdependent (mixed) relationships.

In Differential Calculus, when we differentiate products or composite expressions, we also consider:

- Direct (vertical) contributions — like differentiating one term directly,
- Crosswise effects — how one variable’s change affects the other.

Thus, the Urdhva–Tiryagbhyām Sutra conceptually aligns with:

- the Product Rule
- the Chain Rule
- and Partial Differentiation (in multivariable calculus).

Lopanasthapanabhyam Sutra

The Lopanasthapanabhyam Sutra, meaning “by elimination and retention” or “by omission and restoration”, is an important concept in Vedic Mathematics that emphasizes simplifying expressions by eliminating unnecessary terms and retaining only the essential components. In the study of ordinary differential equations (ODEs), this

sutra is applied to reduce the complexity of equations by systematically removing redundant terms and focusing on the dominant terms that govern the behaviour of the solution.

In ordinary differential equations, Lopanasthapanabhyam is particularly useful when dealing with polynomial differential equations and linear ODEs with constant coefficients, where certain terms vanish during differentiation or integration. By identifying and eliminating these zero-contributing terms, the sutra enables a more efficient formation and simplification of differential equations.

Thus, the Lopanasthapanabhyam Sutra provides a concise and logical approach for simplifying ordinary differential equations, improving clarity and reducing computational effort while maintaining mathematical accuracy.

This sutra is traditionally used in Vedic Mathematics for solving simultaneous equations you eliminate one variable and substitute the result into another equation to find the remaining unknowns.

Application of Vedic Sutras in Different Differential Equation with Illustrative Examples

Application of Vedic Mathematics sutras in ordinary differential equation are explained with examples:

Application of Calana-Kalanabhyam Sutra

Example 1: Derivative of a Polynomial Function

Find $\frac{d}{dx} 3x^4 + 5x^2 - 6x + 9$

Solution

Apply change (Calana) term-wise: $\frac{d}{dx} 3x^4 = 12x^3$

$$\frac{d}{dx} 5x^2 = 10x$$

$$\frac{d}{dx} -6x = -6$$

$$\frac{d}{dx} 9 = 0$$

$$\therefore \frac{d}{dx} 3x^4 + 5x^2 - 6x + 9 = 12x^3 + 10x - 6$$

Example 2: Application to Ordinary Differential Equation

Solve: $\frac{dy}{dx} = 6x^2$

Solution

Using reverse motion (integration): $y = \int 6x^2 dx = 6 \frac{x^3}{3} = 2x^3 + C$

The sutra helps in identifying the change and reconstructing the original function.

The Calana–Kalanābhyām Sutra provides a natural and intuitive approach to differentiation by focusing on rate of change. It simplifies calculations in polynomials, differential equations and enhances conceptual clarity in calculus.

Application of Ekanyunena Purvena Sutra:

Example: Formation of an Ordinary Differential Equation

Consider the function $y = x^4$

Step 1: First derivative (Ekanyunena principle)

$$\frac{dy}{dx} = 4x^3$$

Here, the power reduces from 4 to 3 (*one less than the previous one*).

Step 2: Second derivative

$$\frac{d^2y}{dx^2} = 12x^2$$

Again, the power reduces from 3 to 2.

Step 2: Third derivative

$$\frac{d^3y}{dx^3} = 24x$$

Again, the power reduces from 2 to 1.

Step 4: Fourth derivative

$$\frac{d^4y}{dx^4} = 24$$

Power reduces from 1 to 0.

Formation of the Ordinary Differential Equation

Eliminate the arbitrary constant by forming the ODE:

Since

$$\frac{d^4y}{dx^4} = 24$$

the ordinary differential equation is

$$\frac{d^4y}{dx^4} - 24 = 0$$

Thus, the Ekanyunena Purvena Sutra provides a systematic and efficient approach to determining successive derivatives in ordinary differential equations by reducing powers stepwise, thereby simplifying the formulation and solution of higher-order ODEs.

Application of Gunakamuccayah Sutra:

Example 1: Verification of Factorized Differential Operator

Consider the second-order linear ODE: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Step 1: Write the auxiliary equation

$$m^2 - 5m + 6 = 0$$

Step 2: Factorization

$$(m - 2)(m - 3) = 0$$

Step 3: Apply Gunakasmuccayah Sutra

- Sum of coefficients of $m^2 - 5m + 6$ is: $1 - 5 + 6 = 2$
- Product of sums of coefficients of factors: $(1-2)(1-3) = (-1)(-2) = 2$

Both are equal, confirming correct factorization

Solution of ODE

$$m = 2,3 \quad y = C_1 e^{2x} + C_2 e^{3x}$$

Application of Urdhva-Tiryagbhyam Sutra:

The Urdhva-Tiryagbhyām Sutra, meaning “*Vertically and Crosswise*”, is primarily known for its use in multiplication and factorization. In the context of ordinary differential equations (ODEs), this sutra is effectively applied in the formation, expansion, and factorization of auxiliary (characteristic) equations and differential operators, thereby simplifying the solution process.

Conceptual Role in ODEs

Linear ordinary differential equations with constant coefficients are often solved using auxiliary equations, which involve polynomial multiplication and factorization. The Urdhva-Tiryagbhyām Sutra enables:

- Rapid multiplication of polynomial factors,
- Systematic expansion of differential operators,
- Verification of characteristic equations.

Example 1: Expansion of Differential Operator

Consider the differential equation:

$$(D-2)(D-3)y = 0$$

Apply Urdhva-Tiryagbhyām Sutra (Vertically and Crosswise)

$$(D-2)(D-3) = D \cdot D + D(-3) + (-2)D + (-2)(-3) = D^2 - 5D + 6$$

Thus, the ODE becomes:

$$(D^2 - 5D + 6)y = 0$$

Application of Lopanasthapanabhyam Sutra in ODE

Consider the function

$$y = x^3 + 5x^2 + 7$$

Step 1: First derivative

$$\frac{dy}{dx} = 3x^2 + 10x$$

Here, the constant term 7 is eliminated since it does not contribute to the rate of change. This elimination of a non-effective term illustrates the Lopanasthapanabhyam Sutra.

Step 2: Second derivative

$$\frac{d^2y}{dx^2} = 6x + 10$$

The term involving x^2 is reduced, while only the significant changing terms are retained.

Step 3: Third derivative

$$\frac{d^3y}{dx^3} = 6$$

All variable terms are eliminated, leaving only the essential constant.

Step 4: Formation of Ordinary Differential Equation

$$\frac{d^3y}{dx^3} - 6 = 0$$

Thus, the Lopanasthapanabhyam Sutra assists in ordinary differential equations by eliminating non-contributing terms and retaining only the effective components, leading to simplified and efficient solutions.

CONCLUSION

The present discussion demonstrates that Vedic Mathematics sutras provide an effective, intuitive and systematic approach to the study of ordinary differential equations. Sutras such as Calana-Kalanabhyam, Ekanyunena Purvena, Gunakasamuccayah, Urdhva-Tiryagbhyam, and Lopanasthapanabhyam establish a strong connection between algebraic structure and calculus-based change, thereby simplifying both the formulation and solution of differential equations.

The Calana-Kalanabhyam Sutra emphasizes the concept of rate of change and links algebraic properties of functions with their differentials. The Ekanyunena Purvena Sutra streamlines successive differentiation by systematically reducing powers, making the formation of higher-order ordinary differential equations straightforward. The Gunakasamuccayah Sutra aids in the verification and simplification of auxiliary equations, ensuring accuracy in factorization and solution of linear differential equations. The Urdhva-Tiryagbhyam Sutra facilitates rapid multiplication and expansion of differential operators, reducing algebraic complexity in solving linear ODEs with constant coefficients. Finally, the Lopanasthapanabhyam Sutra enables efficient simplification by eliminating non-contributing terms and retaining only essential components.

Collectively, these sutras reduce computational effort, enhance conceptual clarity, and improve problem-solving efficiency when compared with conventional methods. Hence, the application of Vedic Mathematics sutras in ordinary differential equations not only simplifies analytical procedures but also provides an integrated and elegant framework for understanding advanced calculus, affirming their relevance and usefulness in higher mathematics education and research.

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