A New Weighted Ratio-Cum-Product Estimator for Estimation of the Finite Population Mean Using Known Coefficient of Variation

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ABSTRACT

This paper proposes a new weighted ratio-cum-product estimator using known coefficient of variation for estimating the finite population mean using two auxiliary variables under Simple Random Sampling Without Replacement (SRSWOR). The bias and mean squared error (MSE) of the proposed estimator are derived up to the first order of approximation. Optimum values of the parameters are obtained by minimizing the MSE. A theoretical comparison with existing estimators is presented along with empirical validation using real data sets. The results reveal that the proposed estimator performs better in terms of efficiency and bias under practical condition

Keywords: Finite population mean; Ratio estimator; Product estimator; Auxiliary variables; Mean squared error; Efficiency.

INTRODUCTION

Let $U = \{U_1, U_2, \dots, U_N\}$ be a finite population consisting of N units. Let Ydenote the study variable and X_1, X_2 be the auxiliary variables associated with each unit U_i, $i = 1,2,\dots,N$. The population mean of the study variable is defined as

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i.$$

A sample of size n is drawn from the population using the Simple Random Sampling Without Replacement (SRSWOR) scheme. The corresponding sample mean is given by

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} Y_i ,$$

which serves as an unbiased estimator of $\bar{Y}$.

The use of auxiliary information has been widely recognized as an effective technique for improving the precision of estimators in survey sampling. Over time, several estimators have been proposed utilizing auxiliary variables under simple random sampling. Notable contributions include the ratio estimator introduced by Olkin (1958), the product estimator by Murthy (1964), and further developments by Goodman (1960) and Singh (1965, 1966, 1967). These foundational works have significantly contributed to the advancement of estimation techniques in finite population sampling.

In recent years, further refinements have been made by incorporating multiple auxiliary variables and weighted approaches. Noteworthy contributions in this direction include Panda and Sen (2018), Panda and Chattapadhyay (2022), and Chattapadhyay, Panda, and Mohanty (2022), who developed improved estimators with enhanced efficiency properties.

Motivated by these developments, the present study examines existing estimators for the population mean using two auxiliary variables and proposes a new weighted ratio-cum-product estimator. The proposed estimator is shown to achieve improved performance in terms of bias and mean squared error (MSE) when compared with the estimators suggested by Panda and Sen (2018), Panda and Chattapadhyay (2022), and Chattapadhyay et al. (2022).

Review of Literature of Existing Estimators:

The variance of the simple mean estimator which is unbiased is given by:

$$V(\bar{y}) = \theta_1 \bar{Y}^2 C_y^2 \qquad (2.1)$$

or

$$V(\bar{y}) = \theta_1 \bar{Y}^2 C_0^2 \ . \qquad (2.2)$$

The classical ratio and product estimators of population mean $\bar{Y}$ are respectively defined by as:

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}} \bar{X} \qquad (2.3)$$

and

$$\bar{y}_p = \frac{\bar{y}}{\bar{X}} \bar{x} \ . \qquad (2.4)$$

The expressions for the Biases and Mean Squared Errors (MSEs) up to the first degree of approximation i.e. $O\left(n^{-1}\right)$, are respectively:

$$B\left(\bar{y}_r\right) = \theta_1 \bar{Y}\left(C_x^2 - \rho_{yx} C_y C_x\right) \qquad (2.5)$$

or

$$B(\bar{y}_r) = \theta_1 \bar{Y} \left[C_1^2 - C_{01}\right] . \qquad (2.6)$$

$$B\left(\bar{y}_p\right) = \theta_1 \bar{Y} \rho_{yx} C_y C_x \qquad (2.7)$$

or

$$B(\bar{y}_p) = \theta_1 \bar{Y} C_{01} \qquad (2.8)$$

$$MSE\left(\bar{y}_r\right) = \theta_1 \bar{Y}^2 \left[C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x\right] \qquad (2.9)$$

or

$$MSE(\bar{y}_r) = \theta_1 \bar{Y}^2 \left[C_0^2 + C_1^2 - 2C_{01}\right] . \qquad (2.10)$$

$$MSE\left(\bar{y}_p\right) = \theta_1 \bar{Y}^2 \left[C_y^2 + C_x^2 + 2\rho_{yx} C_y C_x\right] \qquad (2.11)$$

or

$$MSE(\bar{y}_p) = \theta_1 \bar{Y}^2 \left[C_0^2 + C_1^2 + 2C_{01}\right] , \qquad (2.12)$$

where C_x and C_y are the coefficients of variation of x and y, respectively and $\theta_1 = \frac{N - n}{Nn}$.

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The exponential ratio type and exponential product type estimators due to Bahl and Tuteja (1991) are given as:

$$\bar{y}_{re} = \bar{y}\exp\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \quad (2.13)$$

and $$\bar{y}_{pe} = \bar{y}\exp\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} . \quad (2.14)$$

The Biases and Mean Squared Errors (MSEs), up to the first-degree approximation $O\left(n^{-1}\right)$, are:

$$B\left(\bar{y}_{re}\right) = \theta_1 \bar{Y}\left[\frac{3}{8}C_x^2 - \frac{1}{2}\rho_{yx}C_y C_x\right] \quad (2.15)$$

or $$B\left(\bar{y}_{re}\right) = \theta_1 \bar{Y}\left[\frac{3}{8}C_1^2 - \frac{1}{2}C_{01}\right]. \quad (2.16)$$

$$M\left(\bar{y}_{re}\right) = \theta_1 \bar{Y}^2\left[C_y^2 + \frac{1}{4}C_x^2 - \rho_{yx}C_y C_x\right] \quad (2.17)$$

or $$M\left(\bar{y}_{re}\right) = \theta_1 \bar{Y}^2\left[C_0^2 + \frac{1}{4}C_1^2 - C_{01}\right]. \quad (2.18)$$

$$B\left(\bar{y}_{pe}\right) = \theta_1 \bar{Y}\left[-\frac{1}{8}C_x^2 + \frac{1}{2}\rho_{yx}C_y C_x\right] \quad (2.19)$$

or $$B\left(\bar{y}_{pe}\right) = \theta_1 \bar{Y}\left[-\frac{1}{8}C_1^2 + \frac{1}{2}C_{01}\right]. \quad (2.20)$$

$$M\left(\bar{y}_{pe}\right) = \theta_1 \bar{Y}^2\left[C_y^2 + \frac{1}{4}C_x^2 + \rho_{yx}C_y C_x\right] \quad (2.21)$$

or $$M\left(\bar{y}_{pe}\right) = \theta_1 \bar{Y}^2\left[C_0^2 + \frac{1}{4}C_1^2 + C_{01}\right]. \quad (2.22)$$

A ratio-cum-product estimator was proposed by Singh, M.P. (1967), given by:

$$\bar{y}_{rp} = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right). \quad (2.23)$$

The Bias and Mean Squared Error (MSE) up to the first degree of approximation $O\left(n^{-1}\right)$, are respectively:

$$B\left(\bar{y}_{rp}\right) = \theta_1 \bar{Y}\left[C_1^2 + C_{02} - C_{01} - C_{12}\right] \quad (2.24)$$

and $$MSE\left(\bar{y}_{rp}\right) = \theta_1 \bar{Y}^2\left[C_0^2 + C_1^2 + C_2^2 - 2C_{01} - 2C_{12} + 2C_{02}\right] \quad (2.25)$$

Panda and Sen (2018) proposed a weighted ratio-cum-product estimator for estimating the population mean $\bar{Y}$, expressed as:

$$\bar{y}_{RP}^{*} = \bar{y}\left(W_1\frac{\bar{X}_1}{\bar{x}_1} + W_2\frac{\bar{x}_2}{\bar{X}_2}\right), \quad (2.26)$$

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where W_1 and W_2 are the weights of the ratio and product estimators respectively such that . $W_1+W_2=1$
.

The Bias of this estimator, up to the first degree of approximation, is:

$$B\left(\bar{y}^{*}_{RP}\right)=\theta_1\bar{Y}\left(W_1C_1^2 - W_1C_{01}+W_2C_{02}\right) . \tag{2.27}$$

Similarly, the Mean Squared Error (MSE), up to the first degree of approximation, is:

$$M\left(\bar{y}^{*}_{RP}\right)=\theta_1\bar{Y}^2\left(W_1^2C_1^2+W_2^2C_2^2+C_0^2 - 2W_1C_{01}+2W_2C_{02} - 2W_1W_2C_{12}\right) \tag{2.28}$$

Panda and Chattapadhyay (2022) suggested the following estimators for single-phase sampling:

Ratio-cum-exponential ratio estimator:

$$\bar{y}_{rre}=\bar{y}\frac{\bar{X}_1}{\bar{x}_1}\exp\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1+\bar{x}_1} \tag{2.29}$$

Product-cum-exponential product estimator:

$$\bar{y}_{ppe}=\bar{y}\frac{\bar{x}_2}{\bar{X}_2}\exp\frac{\bar{x}_2 - \bar{X}_2}{\bar{x}_2+\bar{X}_2} . \tag{2.30}$$

The Biases and Mean Squared Errors (MSEs) up to the first degree of approximation $O\left(n^{-1}\right)$ are:

$$B\left(\bar{y}_{rre}\right)=\theta_1\bar{Y}\left(\frac{15}{8}C_{x_1}^2 - \frac{3}{2}\rho_{yx_1}C_yC_{x_1}\right) \tag{2.31}$$

or $$B(\bar{y}_{rre})=\theta_1\bar{Y}\left[\frac{15}{8}C_1^2 - \frac{3}{2}C_{01}\right]. \tag{2.32}$$

$$M\left(\bar{y}_{rre}\right)=\theta_1\bar{Y}^2\left(C_y^2+\frac{9}{4}C_{x_1} - 3\rho_{yx_1}C_yC_{x_1}\right) \tag{2.33}$$

or $$M(\bar{y}_{rre})=\theta_1\bar{Y}^2\left[C_0^2+\frac{9}{4}C_1^2 - 3C_{01}\right] . \tag{2.34}$$

$$B\left(\bar{y}_{ppe}\right)=\theta_1\bar{Y}\left(\frac{3}{8}C_{x_1}^2+\frac{3}{2}\rho_{yx_1}C_{x_1}\right) \tag{2.35}$$

or $$B(\bar{y}_{ppe})=\theta_1\bar{Y}\left[\frac{3}{8}C_1^2+\frac{3}{2}C_{01}\right] . \tag{2.36}$$

$$M\left(\bar{y}_{ppe}\right)=\theta_1\bar{Y}^2\left(C_y^2+\frac{9}{4}C_{x_2}^2+3\rho_{yx_2}C_yC_{x_2}\right) \tag{2.37}$$

or $$M(\bar{y}_{ppe})=\theta_1\bar{Y}^2\left[C_0^2+\frac{9}{4}C_2^2+3C_{02}\right] \tag{2.38}$$

Chattapadhyay, Panda and Mohanty (2022) suggested a set of estimators for finite population mean by using the concept of ratio-cum-product estimator due to Singh (1967a) and the exponential product estimate due to Bahl and Tuteja (1991) are as follows:

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Ratio-cum-exponential product estimator:

$$\bar{y}_{rep} = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\exp\left(\frac{\bar{x}_2 - \bar{X}_2}{\bar{x}_2 + \bar{X}_2}\right). \quad (2.39)$$

Product-cum-exponential ratio estimator:

$$\bar{y}_{per} = \bar{y}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)\exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right) \quad (2.40)$$

Ratio-product estimator:

$$\bar{y}^*_{rp} = \bar{y}\left[\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right)\right]\left[\left(\frac{\bar{x}_2}{\bar{X}_2}\right)\exp\left(\frac{\bar{x}_2 - \bar{X}_2}{\bar{x}_2 + \bar{X}_2}\right)\right] \quad (2.41)$$

The expressions for the Biases and MSEs, up the to first degree of approximation, i.e., $O(n^{-1})$ are respectively.

Ratio-cum-Exponential product estimator

$$B(\bar{y}_{rep}) = \theta_1\bar{Y}\left(C^2_{x_1} - \frac{1}{8}C^2_{x_2} - \frac{1}{2}\rho_{x_1x_2}C_{x_1}C_{x_2} + \frac{1}{2}\rho_{yx_2}C_yC_{x_2} - \rho_{yx_1}C_yC_{x_1}\right) \quad (2.42)$$

or $$B(\bar{y}_{rep}) = \theta_1\bar{Y}\left[C^2_1 - \frac{1}{8}C^2_2 - \frac{1}{2}C_{12} + \frac{1}{2}C_{02} - C_{01}\right]. \quad (2.43)$$

$$M(\bar{y}_{rep}) = \theta_1\bar{Y}^2\left(C^2_y + C^2_{x_1} + \frac{1}{4}C^2_{x_2} - \rho_{x_1x_2}C_{x_1}C_{x_2} + \rho_{yx_2}C_yC_{x_2} - 2\rho_{yx_1}C_yC_{x_1}\right) \quad (2.44)$$

or $$M(\bar{y}_{rep}) = \theta_1\bar{Y}^2\left[C^2_0 + C^2_1 + \frac{1}{4}C^2_2 - C_{12} + C_{02} - 2C_{01}\right]. \quad (2.45)$$

Product-cum-exponential ratio estimator

$$B(\bar{y}_{per}) = \theta_1\bar{Y}\left(\frac{3}{8}C^2_{x_1} - \frac{1}{2}\rho_{yx_1}C_yC_{x_1} - \frac{1}{2}\rho_{x_1x_2}C_{x_1}C_{x_2} + \rho_{yx_2}C_yC_{x_2}\right) \quad (2.46)$$

or $$B(\bar{y}_{per}) = \theta_1\bar{Y}\left[\frac{3}{8}C^2_1 - \frac{1}{2}C_{01} - \frac{1}{2}C_{12} + C_{02}\right]. \quad (2.47)$$

$$M(\bar{y}_{per}) = \theta_1\bar{Y}^2\left(C^2_y + \frac{C^2_{x_1}}{4} + C^2_{x_2} - \rho_{yx_1}C_yC_{x_1} - \rho_{x_1x_2}C_{x_1}C_{x_2} + 2\rho_{yx_2}C_yC_{x_2}\right) \quad (2.48)$$

or $$M(\bar{y}_{per}) = \theta_1\bar{Y}^2\left[C^2_0 + \frac{C^2_1}{4} + C^2_2 - C_{01} - C_{12} + 2C_{02}\right]. \quad (2.49)$$

Ratio-product Estimator

$$B(\bar{y}^*_{rp}) = \frac{3}{2}\theta_1\bar{Y}\left(\frac{5}{4}C^2_{x_1} + \frac{1}{4}C^2_{x_2} - \rho_{yx_1}C_yC_{x_1} + \rho_{yx_2}C_yC_{x_2} - \frac{3}{2}\rho_{x_1x_2}C_{x_1}C_{x_2}\right) \quad (2.50)$$

or $$B(\bar{y}_{rp}^{*}) = \frac{3}{2}\theta_1 \bar{Y}\left[\frac{5}{4}C_1^2 + \frac{1}{4}C_2^2 - C_{01} + C_{02} - \frac{3}{2}C_{12}\right]. \tag{2.51}$$

$$M\left(\bar{y}_{rp}^{*}\right) = \theta_1 \bar{Y}^2\left(C_y^2 + \frac{9}{4}C_{x_1}^2 + \frac{9}{4}C_{x_2}^2 - 3\rho_{yx_1}C_yC_{x_1} + 3\rho_{yx_2}C_yC_{x_2} - \frac{9}{2}\rho_{x_1x_2}C_{x_1}C_{x_2}\right) \tag{2.52}$$

or $$M(\bar{y}_{rp}^{*}) = \theta_1 \bar{Y}^2\left[C_0^2 + \frac{9}{4}C_1^2 + \frac{9}{4}C_2^2 - 3C_{01} + 3C_{02} - \frac{9}{2}C_{12}\right]. \tag{2.53}$$

Proposed Weighted Ratio-Cum-Product Estimator:

Utilizing the ideas mooted by Panda and Sen (2018), Das and Panda (2025) and Das, Panda and Sen (2025), we propose a new weighted ratio-cum-product estimator using known coefficient of variation for estimating the population mean $\bar{Y}$, which is expressed as :

$$\bar{y}_{RP_1} = \bar{y}\left[\frac{W_1}{\left(1+\theta_1 C_y^2\right)}\frac{\bar{X}_1}{\bar{x}_1} + W_2\left(1+\theta_1 C_y^2\right)\frac{\bar{x}_2}{\bar{X}_2}\right], \tag{3.1}$$

where C_y is the known coefficient of variation and $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$.

Let $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \Rightarrow \bar{y} = \bar{Y}\left(1+e_0\right)$

Similarly, $e_1 = \frac{\bar{x}_1 - \bar{X}_1}{\bar{X}_1} \Rightarrow \bar{x}_1 = \bar{X}_1\left(1+e_1\right)$

$e_2 = \frac{\bar{x}_2 - \bar{X}_2}{\bar{X}_2} \Rightarrow \bar{x}_2 = \bar{X}_2\left(1+e_2\right)$

Now, $E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = 0$

$E\left({e_0}^2\right) = \theta_1 C_y^2, E\left(e_1^2\right) = \theta_1 C_{x_1}^2, E\left(e_2^2\right) = \theta_1 C_{x_2}^2$

$E\left(e_0 e_1\right) = \theta_1 \rho C_y C_{x_1}, E\left(e_0 e_2\right) = \theta_1 \rho C_y C_{x_2}, E\left(e_1 e_2\right) = \theta_1 \rho C_{x_1} C_{x_2}$

Putting the values of $\bar{y}, \bar{x}_1$ and $\bar{x}_2$ in the equation (3.1), we get

$$\bar{y}_{RP_1} = \bar{Y}\left(1+e_0\right)\left[\frac{W_1}{(1+\theta_1 C_y^2)}\frac{\bar{X}_1}{\bar{X}_1\left(1+e_1\right)} + W_2\left(1+\theta_1 C_y^2\right)\frac{\bar{X}_2\left(1+e_2\right)}{\bar{X}_2}\right] \tag{3.2}$$

$$= \bar{Y}\left(1+e_0\right)\left[W_1(1+\theta_1 C_y^2)^{-1}\left(1+e_1\right)^{-1} + W_2\left(1+\theta_1 C_y^2\right)\left(1+e_2\right)\right].$$

Simplifying the above equation, we can find

$$\bar{y}_{RP_1} = \bar{Y}\left[1 + e_0 + W_2\,\theta_1 C_0^2 - W_1\theta_1 C_0^2 + W_2\theta_1 C_0^2 e_0 + W_1\theta_1 C_0^2\, e_1 - W_1\theta_1 C_0^2 e_1^2\right.$$

$$+ W_2 e_2 - W_1 e_1 + W_1 e_1^2 + W_2 e_0 e_2 - W_1 e_0 e_1 + W_2\theta_1 C_0^2 e_0 e_2 + W_2\theta_1 C_0^2 e_0$$

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$$-W_1\theta_1C_0^2e_0 + W_1e_0e_1{}^2 + W_1\theta_1C_0^2e_0e_1 - W_1\theta_1C_0^2e_0e_1{}^2\Big] . \qquad (3.3)$$

The bias of the proposed estimator, up to the first degree of approximation, is:

$$B\left(\bar{y}_{RP_1}\right) = E\left(\bar{y}_{RP_1} - \bar{Y}\right)$$

$$= E\left(\bar{y}_{RP_1}\right) - \bar{Y}$$

$$= \bar{Y}E\left[W_2\theta_1C_0^2 - W_1\theta_1C_0^2 + W_1e_1^2 + W_2e_0e_2 - W_1e_0e_1\right]$$

$$= \theta_1\bar{Y}\left[-W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02}\right] . \qquad (3.4)$$

Similarly the Mean Squared Error to the first degree of approximation, is

$$M\left(\bar{y}_{RP_1}\right) = E\left(\bar{y}_{RP_1} - \bar{Y}\right)^2 = \bar{Y}^2E\left(e_0 + W_2e_2 - W_1e_1\right)^2$$

$$= \bar{Y}^2E\left[e_0^2 + W_2^2e_2^2 + W_1^2e_1^2 + 2W_2e_0e_2 - 2W_1e_0e_1 - 2W_1W_2e_1e_2\right]$$

$$= \theta_1\bar{Y}^2\left[C_0^2 + W_1^2C_1^2 + W_2^2C_2^2 - 2W_1C_{01} + 2W_2C_{02} - 2W_1W_2C_2\right] . \qquad (3.5)$$

To obtain the optimal values of W_1 and W_2. We minimize the mean square error(MSE) subject to the variation in W_1, implying that

$$\frac{\partial MSE\left(\bar{y}_{RP_1}\right)}{\partial W_1} = 0$$

$$\Rightarrow \theta_1\bar{Y}^2\left[2W_1C_1^2 - 2\left(1-W_1\right)C_2^2 - 2C_{01} - 2C_{02} - 2C_{12} + 4W_1C_{12}\right] = 0$$

$$\Rightarrow W_1 = \frac{C_2^2 + C_{01} + C_{02} + C_{12}}{C_1^2 + C_2^2 + 2C_{12}}$$

Thus $W_{10pt} = \dfrac{C_2^2 + C_{01} + C_{02} + C_{12}}{C_1^2 + C_2^2 + 2C_{12}}$ (3.6)

Where $W_{20pt} = 1 - W_{10pt}$

Comparison of Bias

It is observed that the mean squared errors (MSEs) of the weighted ratio-cum-product estimator due to Panda and Sen (2018) and the proposed weighted ratio-cum-product estimator are equal. Hence, a comparison based on bias is undertaken.

The proposed estimator $\bar{y}_{RP_1}$ is less biased than the estimator $\bar{y}_{RP}^*$, if the condition is satisfied.

$$\left|B\left(\bar{y}_{RP_1}\right)\right| \le \left|B\left(\bar{y}_{RP}^*\right)\right|$$

or $\left|\theta_1\bar{y}\left(-W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02}\right)\right| \le \left|\theta_1\bar{Y}\left(W_1C_1^2 - W_1C_{01} + W_2C_{02}\right)\right|$

or $\left|-W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02}\right| \le \left|W_1C_1^2 - W_1C_{01} + W_2C_{02}\right|$ (4.1)

If $W_1C_1^2 - W_1C_{01} + W_2C_{02}$ is positive then

$$-W_1C_1^2 + W_1C_{01} - W_2C_{02} \le -W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02} \le W_1C_1^2 - W_1C_{01} + W_2C_{02} \quad (4.2)$$

In the first case of (4.2),

$$-W_1C_1^2 + W_1C_{01} - W_2C_{02} \le -W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02} \quad (4.3)$$

After simplification, condition (4.3) reduces to:

$$W_1 \le \frac{C_0^2 + 2C_{02}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)}, \quad (4.4)$$

and $W_2 \ge \dfrac{C_0^2 - 2C_1^2 + 2C_{01}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)}$ (4.5)

where $W_1 + W_2 = 1$.

In the second case of (4.2)

$$-W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02} \le W_1C_1^2 - W_1C_{01} + W_2C_{02} \quad (4.6)$$

After simplification, the condition (4.6) reduces to

$C_0^2 \ge 0$ (Always exists) ,

and $W_1 \ge \frac{1}{2}$, $W_2 \le \frac{1}{2}$

where $W_1 + W_2 = 1$.

If $W_1C_1^2 - W_1C_{01} + W_2C_{02}$ is negative then

$$W_1C_1^2 - W_1C_{01} + W_2C_{02} \le -W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02} \le -W_1C_1^2 + W_1C_{01} - W_2C_{02} \quad (4.7)$$

In the first case of (4.7),

$$W_1C_1^2 - W_1C_{01} + W_2C_{02} \le -W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02} \quad (4.8)$$

After simplification, the condition (4.8) reduces to:

$C_0^2 \ge 0$ (Always exists) ,

and $W_1 \ge \frac{1}{2}$, $W_2 \le \frac{1}{2}$.

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In the second case of (4.7)

$$-W_1C_0^2 + W_2C_0^2 + W_1C_1^2 - W_1C_{01} + W_2C_{02} \le -W_1C_1^2 + W_1C_{01} - W_2C_{02} \qquad (4.9)$$

After simplification the condition (4.9) reduces to:

$$W_1 \ge \frac{C_0^2 + 2C_{02}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)} \quad , \qquad (4.10)$$

and

$$W_2 \le \frac{C_0^2 - 2C_1^2 + 2C_{01}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)} \quad . \qquad (4.11)$$

Comparison of Efficiency

The proposed weighted ratio-cum-product estimator $\bar{y}_{RP_1}$ is more efficient than the existing estimators if the following conditions are satisfied :

$$M\left(\bar{y}_{RP_1}\right) < M\left(\bar{y}_{re}\right) \text{iff} \frac{\left(C_2^2 + C_{01} + C_{02} + C_{12}\right)^2}{C_1^2 + C_2^2 + 2C_{12}} > C_2^2 - \frac{1}{4}C_1^2 + C_{01} + 2C_{02} \qquad (5.1)$$

$$M\left(\bar{y}_{RP_1}\right) < M\left(\bar{y}_{pe}\right) \text{iff} \frac{\left(C_2^2 + C_{01} + C_{02} + C_{12}\right)^2}{C_1^2 + C_2^2 + 2C_{12}} > \frac{3}{4}C_2^2 + C_{02} \qquad (5.2)$$

$$M\left(\bar{y}_{RP_1}\right) < M\left(\bar{y}_{rre}\right) \text{iff} \frac{\left(C_2^2 + C_{01} + C_{02} + C_{12}\right)^2}{C_1^2 + C_2^2 + 2C_{12}} > C_2^2 + 3C_{01} - \frac{9}{4}C_1^2 + 2C_{02} \qquad (5.3)$$

$$M\left(\bar{y}_{RP_1}\right) < M\left(\bar{y}_{ppe}\right) \text{iff} \frac{\left(C_2^2 + C_{01} + C_{02} + C_{12}\right)^2}{C_1^2 + C_2^2 + 2C_{12}} > \left(-\frac{5}{4}C_2^2 + C_{02}\right) \qquad (5.4)$$

$$M\left(\bar{y}_{RP_1}\right) < M\left(\bar{y}_{rp}^*\right) \text{iff} \frac{\left(C_2^2 + C_{01} + C_{02} + C_{12}\right)^2}{C_1^2 + C_2^2 + 2C_{12}} > 3C_{01} + \frac{9}{2}C_{12} - \frac{9}{4}C_1^2 - \frac{5}{4}C_2^2 - C_{02} \qquad (5.5)$$

Empirical Investigation:

To assess the performance of the proposed weighted ratio-cum-product estimator, two empirical studies have been conducted using well-known real-life data sets. The biases, mean squared errors (MSEs), and percentage relative efficiencies (PREs) of the proposed and competing estimators are computed and compared.

Example1:

(Source: Gujarati, 2003, p.269)

In this dataset, the study variable and auxiliary variables are defined as follows:

Y: Annual sales in MPF (Million Paired Feet)

X_1 : Gross National Product (GNP) in billion

X_2 : Unemployment rate (in percent)

The population values are:

N = 16, n = 3, $\bar{Y} = 7543.125$, $\bar{x}_1 = 1287.044$, $\bar{x}_2 = 6.4125$,

$\rho_{yx_1} = 0.2102$, $\rho_{yx_2} = -0.2582$

$\rho_{x_1x_2} = 0.7259$, $C_y^2 = 0.0434$, $C_{x_1}^2 = 0.0289$

$C_{x_2}^2 = 0.1207$, $C_y = C_0 = 0.2083$, $C_{x_1} = C_1 = 0.17$, $C_{x_2} = C_2 = 0.3474$

$C_{01} = \rho_{yx_1} C_y C_{x_1} = 0.0074$, $C_{02} = \rho_{yx_2} C_y C_{x_2} = -0.0187$, $C_{12} = \rho_{x_1x_2} C_{x_1} C_{x_2} = 0.0429$.

$$W_{1opt} = \frac{C_2^2 + C_{01} + C_{02} + C_{12}}{C_1^2 + C_2^2 + 2C_{12}} = 0.6470,$$

$W_{2opt} = 0.3530$ as $W_{1opt} + W_{2opt} = 1$.

Now, we check whether,

$W_1C_1^2 - W_1C_{01} + W_2C_{02}$ is positive or negative .

Here, $W_1C_1^2 - W_1C_{01} + W_2C_{02} = 0.0073$, which is positive

then, $\frac{C_0^2 + 2C_{02}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)} = 0.9375$,

and $W_1 = 0.6470$.

Now, the condition $$W_1 \le \frac{C_0^2 + 2C_{02}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)}$$

is satisfied.

And the other condition

$$W_2 \ge \frac{C_0^2 - 2C_1^2 + 2C_{01}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)}$$

is also satisfied ,

as here, $$\frac{C_0^2 - 2C_1^2 + 2C_{01}}{2\left(C_0^2 - C_1^2 + C_{01} + C_{02}\right)} = 0.0625,$$

and $W_2 = 0.3530$.

The biases of the competing estimators have been computed and presented in Table-1.

Table-1: Biases of the Competing Estimators

Sl.No.	Competing Estimators	$\lvert Bias \rvert / \theta_1 \bar{Y}$
1	$\bar{y}$	0.00001
2	$\bar{y}_r$	0.0215
3	$\bar{y}_p$	0.0187
4	$\bar{y}_{rp}$	0.0401
5	$\bar{y}_{rre}$	0.0431
6	$\bar{y}_{ppe}$	0.0381
7	$\bar{y}^*_{RP}$	0.0073
8	$\bar{y}_{RP_1}$	0.0055

From Table 1, it is evident that the proposed estimator $\bar{y}_{RP_1}$ has the minimum bias among all competing estimators, including the estimator due to Panda and Sen (2018).

The MSEs and the Percentage Relative Efficiencies of the competing estimators presented in Table 2.

Table 2: MSEs and Percentage Relative Efficiencies (PREs) of the competing Estimators w.r.t ($\bar{y}$)

Sl.No.	Competing Estimators	$MSE / \theta_1 \bar{Y}^2$	Percentage Relative Efficiencies
1	$\bar{y}$	0.0434	100
2	$\bar{y}_r$	0.0574	75.610
3	$\bar{y}_p$	0.1260	34.444
4	$\bar{y}_{re}$	0.0432	100.463
5	$\bar{y}_{pe}$	0.0545	79.633
6	$\bar{y}_{rre}$	0.0860	50.465
7	$\bar{y}_{ppe}$	0.2579	16.828
8	$\bar{y}^*_{rp}$	0.1086	39.963
9	$\bar{y}_{RP_1}$	0.0281	154.448

It is clearly observed that the proposed estimator exhibits the highest percentage relative efficiency, indicating its superior performance over all existing estimators.

Example 2:

(Source: Gujarati, p.819)

In this example, data on three economic variables from U.S. time series (1981–83) are considered:

y : Gross domestic product (GDP) in billion $

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x_1 : Personal Consumption Expenditure (PCE) in billion $

x_2 : Personal Disposable Income (PDI) in billion $

The population quantities are:

N = 10, $\bar{Y} = 3803$, $\bar{X}_1 = 2508.28$

$\bar{X}_2 = 2817.53$, $\rho_{yx_1} = 0.1599$, $\rho_{yx_2} = -0.0481$,

$\rho_{x_1x_2} = 0.8727$, $C_y^2 = 0.00018$, $C_{x_1}^2 = 0.00031$,

$C_{x_2}^2 = 9.7214$, $C_{x_1} = C_1 = 0.018$, $C_{x_2} = C_2 = 3.118$,

$C_y = C_0 = 0.013$, $C_{01} = \rho_{yx_1} C_y C_{x_1} = 0.00004$

$C_{02} = \rho_{yx_2} C_y C_{x_2} = -0.002$

$C_{12} = \rho_{x_1x_2} C_{x_1} C_{x_2} = 0.049$.

Using the given data set, the MSEs and Percentage Relative Efficiencies (PREs) of the competing estimators are computed and presented in Table 3.

Table 3: MSEs and Percentage Relative Efficiencies (PREs) of the competing estimators w.r.t ($\bar{y}$)

Sl.No.	Competing Estimators	$MSE/\theta_1\bar{Y}^2$	Percentage Relative Efficiencies
1	$\bar{y}$	0.00018	100
2	$\bar{y}_r$	0.00041	43.902
3	$\bar{y}_p$	0.00026	69.231
4	$\bar{y}_{re}$	0.00021	85.714
5	$\bar{y}_{pe}$	0.00029	62.069
6	$\bar{y}_{rre}$	0.00076	23.684
7	$\bar{y}_{ppe}$	0.00038	47.368
8	$\bar{y}_{rp}^*$	21.6475	0.000832
9	$\bar{y}_{RP_1}$	0.00012	150

From Table 3, it is again evident that the proposed estimator achieves the highest efficiency among all competing estimators.

Computation of Percentage Relative Efficiency (PRE)

The Percentage Relative Efficiency (PRE) of an estimator is computed using the following expression:

$$\text{Percentage Relative Efficiency} = \frac{\text{MSE of the Base estimator } (\bar{y})}{\text{MSE of the Estimator}} \times 100$$

Here, the Base is taken as the sample mean (mean per unit estimator), denoted by $\overline{y}$.

CONCLUSION

The proposed weighted ratio-cum-product estimator effectively utilizes dual auxiliary information to improve the estimation of the finite population mean. The theoretical expressions for bias and MSE demonstrate that the estimator is more efficient under practical conditions. Empirical results further support the superiority of the proposed estimator over classical estimators.

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