Caputo-Type Fractional Differential Equations: Recent Advances and Review

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ABSTRACT

This review examines recent advances in Caputo-type fractional differential equations, focusing on theoretical foundations, existence and uniqueness results, and progress in numerical methods. Particular emphasis is placed on discretization schemes, convergence analysis, and high-order hybrid techniques that enhance accuracy and efficiency. Applications in science and engineering such as viscoelasticity, anomalous transport, population dynamics, and epidemiology demonstrate the broad utility of Caputo-type models in capturing memory effects and long-range interactions. The review also explores nonlinear formulations, optimal control, inverse problems, and stochastic extensions, underscoring both the versatility and the challenges of Caputo-based approaches. Finally, the review outlines open questions and future research directions, offering a roadway for advancing fractional calculus theory and applications in complex systems.

Keywords: Caputo fractional derivative; Fractional differential equations; Existence and uniqueness; Numerical methods; Viscoelasticity; Anomalous diffusion; Population dynamics; Epidemic modeling; Optimal control; Stochastic extensions

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INTRODUCTION

The Renewed Relevance of Caputo-Type Models

Caputo-type (C) models enable legislators to gauge the effect of interventions on epidemics in the short, medium, and long term. They are also overseen by well-established methodologies such as optimal control (Almeida ,., 2015) or inverse problems (Wu, 2017), which allows the parameterization and identification of systems currently known to bear physical interpretation. (Baghel, 2026) (Olayiwola & Abiodun, 2025) (Rashid ,., 2026) (Barman ,., 2024) (Baleanu & Alshomrani, 2022) (Orosco, 2023). Considerable research has addressed nonlinear extensions of C systems, relating to bifurcation (J. Lazo & F. M. Torres, 2012) or long-time behaviour (André ,., 2018). Nevertheless, the corresponding existence and uniqueness questions remain to be clarified, along with insights into their attractors and the stability theory they engender. (Shang ,., 2026) (Chen ,.2025) (Vignesh ,., 2025) (Stoychev & Römer, 2023) (Lin ,.2025) (Del & Flandoli2024) (Ashhab ,., 2026) (Agresti &Veraar2025) (Zahid ,., 2024). C models are applied to science and engineering, yet much remains to be explored in practice and in theory in relation to nonclassical derivatives and the associated C models. Population dynamics (André ,., 2018) occurs on the edge of viability, in particular due to incomplete development of inverse issues. The existing classes of C functionals or functionals in general remain scattered and in need of coalescence. (Kasneci ,.2023) (Liu ,., 2025) (Nguyen ‐ Duc ,.2025) (Stein ,.2024) (Sorscher ,.2022) (Jeon & Lee, 2023) (Dong ,.2024)

Fundamentals Refreshed: Core Definitions and Notions

In a real, bounded vector space X, the Caputo fractional derivative of order $\alpha > 0$ exists. The operator, defined through integration by parts, is of the form $c\,D_t^{\alpha}\varepsilon(t)$, $\varepsilon(t)$ the solution of an evolution equation. A real-valued, bounded kernel K(t,s) satisfying suitable growth conditions determines the evolution operator, which

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may be bounded or even compact. In a space of real-valued functions $K^{\alpha,\tau}$ with $\tau = 0$, the Caputo fractional evolution equation is convex, and solution continuity in $K(0,\infty)$ implies the existence of an equilibrium. Bifurcation phenomena occur when the equilibrium involves multiple spatial variations. (Feng & Sutton, 2020) The Caputo fractional differential equation with distributed order describes a process related to a pure fractional derivative of an unknown function u(s,t) with distributed-order density in a bounded interval. The order of the distributed Caputo derivative is also varied. Distributed-order models reflect phenomena with multiple time scales, operationally equivalent to functional-differential equations or an infinite series. (Mainardi & Gorenflo, 2008) The Caputo fractional derivative, defined through the system of fractional-order conventional derivatives used in materials science, plays a foundational role in machines with built-in stiffness. Test Volterra-based Caputo-type expressions incorporate input frequency and identify in-phase and out-of-phase characteristics of time- and frequency-domain reactions. (Padder .2023)(Amilo .2025)(Nuca & Parsani, 2024)(Ul .2024)(Kosari .2026)(Rayal ., 2023)(Mehmood .2023)(Kubra & Ali, 2023)

Existence and Uniqueness: Modern Perspectives and Techniques

Existence, uniqueness, and continuous dependence results for Caputo-type fractional initial-value problems recently received significant attention, and new advances employ modern fixed-point and monotone operator frameworks (Dahmani, 2015). In addition to classical linearity requirements on the right-hand side, the results allow for potentially unbounded Lipschitz-type assumptions. These and other extensions have considerable implications for phenomena modeled by Caputo equations, including viscoelasticity, anomalous diffusion, and population dynamics. Furthermore, new results for nonlocal fractional problems establish the existence of solutions using monotone graph conditions; these conditions appear in a wide range of applications. (Batit Ozen, 2025)(Lin , 2026)(Awad ,., 2023)(Albasheir ,.2023)(Madamlieva & Konstantinov, 2025)(Hu , 2026)(SIVARANJANI .)(Udo & Eze, 2025)(Ennaceur .2026). Given the generality of the above framework, model-relevant conditions often emerge depending on physical parameters and numerical discretizations. For example, Caputo equations with nondecreasing kernels arise naturally in time-fractional Cattaneo and telegraph-like heat-transfer models, yet frameworks ensuring well-posedness with very weak restrictions on kernels may not apply. Caputo-type problems with homogeneous-source terms leading to closed dynamic models are frequently encountered in viscoelasticity and elastoplasticity. Furthermore, well-posedness for equations without regularizing convolution kernels remains a critical aspect of literature addressing transport, diffusion, or population- and epidemic-dynamic model Caputo formulations. Natural bifurcation phenomena and long-time dynamics have also accompanied the emergence of new nonlinear models. (Karde ., 2025)(Ibrahim .2026)(Geetha2025)(Kosztołowicz, 2023)(Wei ., 2023)(Alhejail & Alsaadi, 2026)(Ibrahim .2026)(Dekhkonov, 2026)(Angelani .2025)(Bonforte 2026)

Numerical Methods: Advances in Accuracy, Stability, and Efficiency

The Caputo derivative is a commonly used definition of fractional derivative in the mathematical modeling of many real-world phenomena. Clearly separated at an early modeling stage, the memory effects involved in the modeling of the underlying phenomena are fundamental for assessing their theoretical properties, which may influence the choice of numerical methods, meanwhile, offering additional modeling flexibility for boundary conditions. Recall in agreement with the Riemann-Liouville derivative and a widely distributed-theoretic generalization definition for fractional derivatives alike, the Caputo derivative is an operator that enables to set initial conditions constituted by usual integer derivatives without additional specifications. Noteworthy properties such as with respect to nonlocal, continuous dependence on initial data and the attraction of the zero equilibrium with regard to autonomous systems remain well-documented in literature. In the Caputo context, classical models having no fractional term, like ordinary differential equations or partial differential equations, at present have achieved significant advancements since the early works on this topic. Such studies brought to attention several research topics on Caputo fractional differential equations computable by numerical and analytic methods, inclu336328b1-bb18-4fbd-9638-244ff96cfc8c the existence, uniqueness, and finite-time blow-up, bifurcation and chaotic dynamics, high-order and stabilizing numerical schemes; and more recently, numerous contributions on the modeling and analysis of applications of Caputo equations (Aceto & Novati, 2018) (Tavares., 2015) (Ding & Li, 2014). (Nuca & Parsani, 2024)(Saadeh .2024)(Ul2024)(Khurshaid & Khurshaid, 2023)(Padder.2023)(Alshehry l., 2024)(Altan ., 2023)(Deppman., 2023)

Discretization Schemes for Caputo Derivatives

Many numerical methods have been proposed for fractional differential equations, focusing mostly on the Riemann-Liouville derivative. Caputo derivatives can also be considered through alternative formulations and schemes based on the Riemann-Liouville derivative, notably the Grunwald–Letnikov approximation and convolution quadrature (Tavares, 2015). A predictor–corrector scheme has been developed via a semi-implicit discretization of the Caputo derivative. Such a scheme preserves the stability region associated with a problem's initial value. Convergence rates of these timespace discretization methods have also been examined, enabling an understanding of critical regularity requirements. An interesting coupling with different macroscopic partial differential equations has been explored that yields a multi-physics framework. Fractional calculus frequently employs classical polynomial approximations such as finite-element or finite-difference methods. Consequently, spectral and spectral-collocation methods have been introduced to solve initial value problems involving the Caputo derivative. (Tobita & Matsuo, 2023)(Bu & Jeon, 2025)(Kangdi , 2025)(Shen , 2023)(Ndengna, 2024)(Wang .2025)(Sebastiano & Izzo, 2026)(Toutlini , 2024)(Schörghofer & Khatiwala, 2024). The Caputo derivative of order $\alpha \in (0, 1)$ of the function $u(t)$ is defined by a convolution of the Riemann–Liouville fractional integral and the classical derivative: (Rahman ,.2025)(Nuca & Parsani, 2024)(Atangana, 2025)(Nosheen ,.2024)(Moschandreou2024)(de Oliveira & Maiorino, 2024)(Atangana & Koca2025)

$$c\ D^{\alpha}{}_0\ u(t) = {}_0D^{\alpha}{}_0 I_0{}^{1-\alpha}\ u(t)\ , \quad t>0$$

with the Caputo fractional derivative $c\ D^{\alpha}{}_0\ u(t) = {}_R\ D^{\alpha}{}_0\ (u(t) - u(0))$

and its representation in integral form is given as follows:

$$c\ D^{\alpha}{}_0\ u(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha}\, u^{(1)}(s)ds$$

The non-singular derivatives formulated by Caputo–Fabrizio define a new type of fractional derivative and can be numerically solved with the help of operator splitting techniques. Time-discontinuous Galerkin and continuous methods have been used for Caputo–Fabrizio derivatives. The Caputo–Fabrizio operator improves convergence from $(O(\tau^{2-\alpha}))\ to\ (O(\tau^{3-\alpha}))$

in a discretized fractional transport equation. Boundary–value problems with a generalized Riemann–Liouville and a Caputo–Fabrizio are addressed using finite difference solutions for which convergence and stability properties are proved. (Mohammed & Rafeeq2024)(Noori Dalawi ,., 2023)(Kumar ., 2026)(Parvin .2025)(Ibrahim & AlSheikh, 2026)(Nasr.2026)(Dozva .2026)(Agilan & Parthiban, 2026)

Convergence Analysis and Error Estimates

Convergence estimates accompanying numerical schemes for the Caputo-time fractional differential equation are reported in (A. Alikhanov & Huang, 2021) , (Davis , 2018) , and (Jin , 2015). A scheme approximating the Caputo derivative on a uniform mesh achieves order $O(\tau^{3-\alpha})$ with time discretization k=O(τ). Stability with Lipschitz continuous solutions is established by energy methods compatible with extension to nonlocal operators. A first-order L1 time-stepping scheme on a timedependent two–dimensional domain preserves the semigroup structure of linear problems. Rate of convergence for regularities less than C^2 is obtained, bolstering previous results on equation-dependent sectorial operators, including space-time fractional equations, and extending to nonuniform meshes. Convergence estimates for the one-dimensional Caputo–time fractional diffusion equation with nonconstant diffusion coefficients remain open. (Kosztołowicz, 2023)(Rodrigues, 2026)(Momani ,.2026)(DURDIEV & TURDIEV, 2027)(Mohamed & Qamlo, 2026)(Guo ,., 2026). Convergence estimates for caputo-type differential equations are documented in;;. Standard methods yielding schemes for caputo derivative discretization on uniform grids are analysed; convergence orders and required solution regularity are specified. A caputo-type fractional differential equation with nonlocal conditions is investigated; first-order convergence of an L1-type scheme on an adaptive timediscretization is shown for sufficiently smooth solutions and segmentwise constant temporal meshes. A gerasimov–caputo evolution

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transmission problem with dirichlet–neumann conditions is considered; conversion into the equivalent integro-differential form leads to the study of a numerical approach for a corresponding space–time fractional diffusion model containing a caputo fractional derivative; convergence results remain unavailable. (Demir & Tosunoğlu, 2025)(Albadarneh .2025)(Allogmany & Alzahrani, 2025)(Pitti, 2025)(Peng , 2025)(Ou ., 2024)(Amilo 2025)(Nguyen .2026)(Tverdyi2025)

High-Order and Hybrid Methods

High-order and hybrid methods improve the efficacy of Caputo-type models by leveraging spectral geometric and adaptive techniques. Spectral collocation methods reduce computation times compared with Galerkin techniques while preserving accuracy, whereas adaptive schemes introduce variations to the temporal discretization for additional efficiency. Multi-physics coupling enables reduced-order models by decoupling different physical processes, permitting solution of the Caputo-type model alongside other governing equations of the system (Tavares., 2015). Together, these approaches form a coherent strategy for both improving accuracy and reducing effort, addressing the first two criteria in the hierarchy of sought-after numerical properties while retaining fidelity to the underlying phenomena (Chen & Deng, 2013). (Shams & Carpentieri, 2026)(Alhazmi , 2025)(Abuasbeh & Arab, 2026)(Kheybari , 2025)(Ilati & Eslami2026)(Ali khanov .2025)(Derakhshan & Irandoust Pakchin, 2026)(Kandwal & Patel, 2025)(Khan 2026)

Applications in Science and Engineering: A Broader Impact

Descriptions of phenomena involving memory effects abound in science and engineering. Constitutive equations in viscoelasticity, for example, relate stress and strain through a history integral, where the relaxation kernel describes how the effect of past deformations influences the current stress. The study of anomalous transport is motivated by irregular migratory behavior in ecology and finance; spatial models of non-standard diffusion have memory-based representations, with temporal kernels modulating the influence of past concentrations on future evolution. Population dynamics with long-range interactions take the form of integral equations in which an average of past densities determines present growth. In epidemiology, fractional models formalize the delay associated with account-opening procedures, and also describe memory arising from multiple routes of infection. The order of the fractional operator carries an interpretation–the degree of memory in the system. (Zhang , 2023)(Li)(Górska & Horzela, 2023)(Liu & Geng, 2024). Fractional differential equations governed by the Caputo derivative have been used to describe transport phenomena with memory (Almeida ,., 2015). Such models arise in viscoelasticity, where particle displacements depend on past deformations and early-time behaviour corresponds to a standard material. In subdiffusion and superdiffusion, Caputo-based equations relate time-integral averages of past densities to present concentrations, extending classical models. In population dynamics, memory-driven growth leads to the consideration of fractional Caputo models that accommodate different interpretations about whether past values or increments are averaged. (Cusseddu, 2026)(Ghezal ,., 2025)(Salama & Fairag, 2024)(Kumari ., 2024)(Ashok & Sayed2024)(Al-Mekhlafi2026)(Hugh & Soria2025). Caputo-type fractional differential equations with real-order derivatives formalize widely-recognized phenomena and provide clear interpretations of the associated order. Well-posedness theory, numerical methods, nonlinearity, control, stochastic extensions, and other avenues of study serve to advance scholarship in this area. (KEBEDE, 2025)(Khan .2025)(Kang .2026)(Wang .2024)(Aydin & Mahmudov, 2025)(Ghosh & Kumar, 2026)(Shams & Carpentieri, 2025)(Rashid , 2026)(Glaeser .2023)

Viscoelasticity and Material Science

In viscoelasticity and material science, the analysis of fractional differential equations and their applications has gained increasing attention (Amirian Matlob & Jamali, 2017). Fractional calculus is used in constitutive equations to model dissipation and damping phenomena in viscoelastic materials, while multi-term fractional differential equations with Caputo derivatives are employed in fractional-order and/or multi-scale viscoelastic models. Experimental results demonstrate that, in polymers and polymeric composites containing carbon black, the quality factors Q obtained from damping data exhibit nearly frequency-independent behavior, indicating the importance of fractional-order derivatives in these dissipative/damping phenomena. Fractional derivatives also provide an alternative formulation for the linear fractional-order differential equation governing the motion of a viscoelastically damped structure and in modeling hysteretic behavior in electromagnetic devices. In both cases, the Caputo fractional derivative is preferable, as experimental results can be interpreted in terms of material properties without the need for polynomial approximations of the operational transfer function. (Bhangale .2023)(Ding., 2026)(Alghamdi, 2025)(Ferrás, 2025)(Wang & Wang, 2023)(Han .2024)(Abouelregal.2026). Analytical and numerical methods for solving multi-term fractional differential equations with Caputo derivatives have been applied to a large variety of linear and non-linear problems. Numerous numerical approaches, including finite difference, finite element, and Galerkin methods, have been developed to handle multi-term fractional differential equations arising in viscoelastic materials. (Lin ., 2023)(Arifeen ., 2024)(Fan, 2023)(Brandibur & Kaslik, 2023)(Abd-Elhameed & Alsuyuti, 2023)(Zou & Zhang, 2025)(Santra, 2025)(Khosravian-Arab & Dehghan2024). While ordinary differential equations can capture long-range memory effects in the dynamics of complex systems possessing remanent and/or delayed memory, multi-term fractional differential equations can represent complex systems with both short-range and long-range memory, enabling the consideration of longer time delays interspersed with shorter ones. The timescales associated with fractional-order derivatives are independent of the unsigned amplitude of the perturbations, and an increase in the degree of nonlinearity can lead to an enhancement or suppression of delayed bifurcation, depending on the nature of the nonlinearity. (Vasylyeva, 2024)(Yang ,.)(Brandibur & Kaslik, 2023)(ur .2024)(Engström & Morsalfard, 2026)(Fahmy & Marin2026)(Fan,2023)(Lin , 2023)(Santra, 2025)

Anomalous Transport and Diffusion Processes

Anomalous diffusion encompasses processes whose mean-square displacement varies in proportion to a non-integer power of time. Subdiffusion (exponent < 1) manifests as hindered transport in porous media or gels, while superdiffusion (exponent > 1) arises from Lévy flights or inertial particle motion (Machida, 2016). In porous media, correlated random walks of propagating molecules yield fractional-Fokker-Planck models incorporating Caputo derivatives (Liang , 2018). Fractional equations more broadly characterize phenomena such as time-dependent diffusion, early-time transport, cross-diffusion on fractals, and distributed-order kinetic equations (M. Sokolov, 2004). (Zhang 2024)(Deniskin & Estrada, 2026)(Berardi .2025)(Essawy ., 2025)(Deniskin & Estrada, 2026)(El-Nabulsi .2026)(Abouelregaletal.2025)(Almajbri,2025)(He , 2025)

Population Dynamics and Epidemiology

Population dynamics is an area in which Caputo-type fractional derivatives arising from memory effects and long-range interactions can be profitably employed. Classical models are typically based on first- or second-order derivatives, corresponding to instantaneous population growth and aggressive competition, respectively. In contrast, the Caputo-type variant incorporates a memory kernel into the birth function, allowing for historic influences on the recruitment of new individuals. Mathematically, the Caputo derivative can be employed in a straightforward extension to a multi-patch environment. Within an isolated patch, growth is dependent on the present and past number of individuals. Population dynamics problems in a multi-patch domain describe how the population in one location influences the dynamics elsewhere. Both full and multi-patch formulations have subject to thorough numerical analysis. Adopting a different approach, epidemic models have also been extended to the Caputo framework. Individuals are classified into various compartments that determine their level of contagiousness (P. Harris ., 2022). Fractional characteristics, accounted for via the Caputo–Fabrizio derivative, emerge naturally where the population generation is governed by non-local mechanisms or where transmission persists over extended intervals (Boudaoui , 2021). (Kosari .2026)(Bhatter , 2025)(Ullah ., 2023)(Hajaj & Odibat, 2023)(Olayiwola & Yunus2025)(Alshammari..2025)(Prasad , 2023)(Kumar,2023. In a different formulation, the Caputo operator is employed to describe both birth and death processes. This construction exploits the analogy between classical epidemic models and earthquake time-series models, within which the first-order statistics of the two settings exhibit similar scaling properties (Garra & Polito, 2011). The additional flexibility afforded by a semi-infinite interval permits embodied multi-epidemic dynamics. Typical formulations account for temporal dependence through a time-lag mechanism; a Caputo operator naturally provides access to non-exponential memory kernels. (Farman ,., 2023)(Gkrekas, 2024)(Xu . 2024)(Tassaddiq ,2024)(E2024)(Hajaj & Odibat, 2023)(Alshammari ,2025)(Subramanian,2024)

Nonlinear Caputo-Type Problems: Existence, Uniqueness, and Dynamics

Nonlinear Caputo-type problems arise in various mathematical models across science, engineering, and finance. In most formulations, the effects of time-dependent material aging or the delay of competing species in ecology are coupled with nonlinear reactions. Yet the additional complexity introduced in the fractional setting renders poor knowledge and weak results on associated notions of existence, uniqueness, multiplicity, and boundedness of solutions—key ingredients to the accurate assessment of global behavior and long-time dynamics. One novel study examines Caputo-type initial-value problems with time-dependent delays and interaction terms described by a polynomial. It establishes the existence, uniqueness, and boundedness of solutions under suitable assumptions, thereby ensuring well-posedness and providing the foundation to investigate long-time dynamics further (Tisdell , 2017). In a distinct investigation, new global existence and uniqueness results for systems of coupled nonlinear fractional differential equations are presented, establishing the well-posedness of Caputo-type fractional systems with such high-dimensional coupled nonlinearities (Dahmani, 2015). Boundary-value problems for multi-term Caputo-type fractional differential equations—including those with initial conditions, nonlocal boundary conditions, and smile-type conditions linked to bifurcation delays—are also scrutinized, leading to Ulam-type stability and solution properties (Houas & Bezziou, 2019). (Shams .2023)(Shams .,2024)(Lenka & Bora, 2023)(Li ., 2023)(AlBaidani 2024)(KEBEDE,2025)(Lenka & Bora, 2023)(Allogmany & Alzahrani, 2025)(Olayiwola & Yunus2025)(Sivalingam & Govindaraj, 2025).An analysis of the behavior of solutions to Caputo-type fractional differential equations with nonlinear terms reveals the asymptotic convergence or nonconvergence of solutions toward steady states, the existence of periodic and quasi-periodic solutions, and the emergence of bubbling solutions subjected to forcing terms. Specific attractors, such as the global attractor, pullback global attractor, uniform pullback attractor, and H-sets, are identified and their interrelations discussed. The stability of the corresponding Caputo-type incomplete multiscale termites problem in ecology and the effect of time lags on cascading bifurcations in a Caputo-type ratio-dependent predator-prey system are also examined. (Fafa.2023)(Sytnyk & Wohlmuth, 2023)(Shams.2023)(Huong & Anh, 2026)(Kassim, 2024)(KEBEDE, 2025)(Agarwal & Madamlieva, 2025)(Nadeem & Iambor, 2023)(Liu ,.2025)(Kebede .2024)

Control, Optimization, and Inverse Problems

The optimal control theory for systems governed by fractional differential equations (FDEs) has rapidly progressed since 2010, yet Caputo dynamics remain relatively unexplored. To address this gap, optimal control problems involving Caputo FDEs of all orders are formulated. State and control constraints are included, and verification results are established. A Pontryagin maximum principle is derived from first principles, providing necessary conditions that describe how optimal controls can be inferred from the state trajectory. Employing an adjoint method reformulates the problem using adjoint Caputo dynamics, leading to a characterization of the adjoint system. Concrete identifiability results are also provided, illustrating how the structure of the control problem ensures observational openings within the state trajectory that permit the complete recovery of the control. (Abd-Elmonem ,.2023)(Mojahed ,., 2025)(Damak, 2023)(Cao ,., 2023)(Elsonbaty ,., 2025)(Dekhkonov, 2026)(Jiang2025)(Ali ,.2026)(Mohsin ,.2025)(Moon, 2025). Many situations described by EFDs exhibit history-dependence, thus necessitating delayed control problems. Control models with fractional states naturally arise from the synthesis of nonlinear ordinary differential equations and Caputo derivatives. Inverse control problems have been defined for finite-dimensional systems governed by Newell-Whitehead-Segel type Caputo equations. Analytical identifiability results for these equations have been established in the one-dimensional case without spatial domain constraints. The nature and location of control openings synonymous with the observability for systems described by distributed parameter systems (DPS) provide avenues for future work (Pooseh, 2013). (Jiang2025)(Saadeh ,.2024)(Padder ,.2023)(Kebede & Lakoud, 2023)(Kosari ,.2026)(Sadek, 2025)(Mohamed & Qamlo, 2026)(Almutairi & Saber, 2023)

Stochastic Extensions and Uncertainty Quantification

Fractional calculus is successfully employed to describe anomalous stochastic processes that are superior to classical models. Unfortunately, random fractional orders have not yet been incorporated into stochastic Caputo models. An operator-based uncertainty quantification framework for stochastic fractional partial differential equations with additive noise has been developed by treating the fractional orders as uncertain variables (Kharazmi & Zayernouri, 2018). In addition, sensitivity analysis techniques for the stochastic Caputo operator are available, facilitating the investigation of the uncertainty propagation and identification of the most influential parameters. Random-sequence-based Monte Carlo and quasi-Monte Carlo methods for the solution of multi-dimensional stochastic Caputo models are also under development. (Kosari ,.2026)(Almutairi ,.2026)(Padder ,.2023)(Majee ,.2024)(Jan ,.2023)(Kumar ,.2023)(Amilo ,.2025)(Keddi & Bouzebda, 2026)(Rashid ,., 2023)

Challenges and Open Questions: A Roadmap for Future Research

Real-world phenomena occur at different scales and time horizons that classical models based on local behavior cannot handle. Most differential equations governing such phenomena involve integral operators depending on past states—i.e., memory effects. Fractional differential equations, a natural extension of conventional differential equations, incorporate a wide range of memory effects using non-integer derivatives. The Caputo fractional derivative is a well-structured and frequently used definition. The corresponding initial-value problem is of primary interest, as applications typically involve state dynamics with specified initial conditions. Current approaches focus on existence and uniqueness results for the initial-value problem using fixed-point, monotone operator, and variational techniques; numerical methods for high-order discretization in time and space; and a spectrum of applications in the natural, social, and engineering sciences. (Hilal & AlShemmary, 2026)(Khan)(Shams & Carpentieri, 2025)(Abidin, 2026)(Faiz ,., 2026)(Dai ,., 2024)(Akram ,., 2025)(Sumaiya ,.2025)(Raghavendran ,.2025).Existing work on Caputo fractional differential equations cannot fully address real-world phenomena, creating opportunities for further research and significant societal impact. Challenges remain in existence and uniqueness, control, inverse problems, stochastic extensions, and uncertainties (Wu, 2017). Models in certain scientific disciplines, notably field data in water resources and public health, are more complex than Caputo-type models; progress on multidimensional and nonlinear equations is needed (Almeida ,., 2015). A variety of numerical problems arise in coupling Caputo-type equations with finite-volume schemes, particle-in-cell schemes, analytical solvers, and other methods. The influence of initial data and parameters on long-time behavior is also poorly understood. Addressing these issues will benefit from cross-disciplinary collaboration, attention to other fields and models, and the establishment of benchmark examples and test problems. (Ashok & Sayed2024)(Bhatter ,., 2025)(Ali ,.2026)(Baleanu ,.2023)(KEBEDE, 2025)(Amilo ,.2025)(Alshammari ,.2025)(Ahadi ,.2025)(Hammad ,., 2025)

CONCLUSION

Fractional models also invoke jump history and dissipative memory in epidemiology, further supporting the relevance of Caputo equations to pressing issues. By adding these aspects to models with simple delays, researchers depict memory and long-distance interactions reminiscent of animal movement (Almeida ,., 2015). The capacity to reproduce the typical logistic trajectory is particularly valuable for policymakers addressing population growth. Caputo problems arise naturally in situations where classical integer-order formulations fail; thus, theoretical and numerical studies bolster understanding of governing equations and offer comparison benchmarks.

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