Optimal Production and Inventory Policies for Delayed Deteriorating Items with Stock-Dependent Demand and Partial Backlogging under Two-Phase Production: Insights from COVID-19 Disruptions

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ABSTRACT

The COVID-19 pandemic exposed critical weaknesses in global supply chains, necessitating strong and sustainable inventory strategies for perishable goods. This study proposes an economic production quantity (EPQ) model for items subject to delayed deterioration under a two-phase production framework with shortages and partial backlogging. The production process is divided into two distinct phases representing different stages of disruption and recovery. During the production period, demand is assumed constant, whereas in the non-production period, demand depends on the available stock level, reflecting post-pandemic consumer behavior in which product visibility influences purchasing decisions. The model incorporates a finite freshness period, after which deterioration begins, making it suitable for products such as meat, bread, and cassava. Shortages are permitted and partially backlogged to capture realistic market responses. The objective is to determine the optimal production cycle and inventory levels that minimize the total system cost, including production, holding, deterioration, shortage, lost sales, and environmental costs. The purpose is to determine the optimal cycle length and inventory level for each cycle to minimize total cost. The necessary and sufficient conditions are provided to show the existence and uniqueness of the optimal solution. Also, the decision rule of the Newton-Raphson method has been used to determine the optimal solutions for the nonlinear optimization problem. Numerical illustrations and sensitivity analyses are presented to examine the impact of key parameters, including demand sensitivity, deterioration rate, and backlog fraction, on optimal decisions. The results demonstrate that the proposed framework provides practical insights for manufacturers seeking to stabilize operations, reduce waste, and enhance profitability in the post-COVID-19 recovery phase.

Keywords—*EPQ,* delayed deterioration, stock-dependent demand, partial backlogging, two-phase production, COVID-19

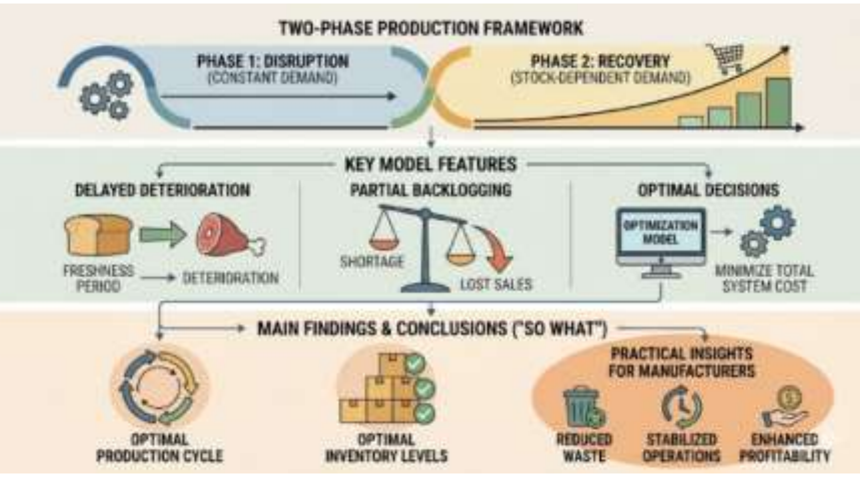

Figure 1. Graphical Abstract

Highlight of the graphical abstract

A two-phase EPQ model integrates delayed deterioration with stock-dependent demand for post-pandemic supply chain resilience

Partial backlogging captures realistic customer waiting behavior during shortage periods under disruption conditions

Environmental cost inclusion significantly influences optimal production cycles and reduces spoilage-related waste

INTRODUCTION

The COVID-19 pandemic caused severe disruptions in production and logistics worldwide. Perishable items, such as meat, bread, and cassava, are particularly vulnerable due to their limited shelf life. Traditional EPQ models often fail to consider delayed deterioration, stock-dependent demand, and partial backlogging, which are crucial for designing resilient post-pandemic supply chains.

Maintenance of inventories of deteriorating items is a problem in the supply chain of almost any business organization. One of the most unrealistic assumptions in traditional inventory models is that items preserve their physical characteristics while being kept in inventory. Generally, deterioration is the physical depletion/decay of products over time, which prevents an item from being used for its purpose. In general, almost all products are found to deteriorate over time. Sometimes the rate of deterioration is too low for items such as hardware, glassware, metals, and toys; however, some items have a significant rate of deterioration, such as perishable foodstuffs, vegetables, gasoline, chemicals, and so on. This phenomenon cannot be ignored in the decision-making process of production lot size. [1] first studied an EPQ model for deteriorating items with varying and constant rates of deterioration; others are [2], [3], [4], [5], and so on, with the assumption that demand rate, production rate, and deterioration rate are all constant. [6] Considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates. In all the models stated above, shortages are not allowed. However, [7] established a production inventory model with constant production rate, demand rate, and deterioration rate and allowed shortages. [8] Also considered a production inventory problem in which each cycle of a production inventory schedule starts with replenishment and ends with a shortage.

The primary operation strategies and goals of most manufacturing firms are to seek a high satisfaction of customers' demands and to become a low-cost producer. To achieve these goals, the company must be able to effectively utilize resources and minimize costs. The economic production quantity (EPQ) model is commonly used by practitioners in the fields of production and inventory management to assist them in making a decision on production lot size.

Inventory management has remained a major area of study in operations research because of its role in balancing cost, service level, and production efficiency. Classical models such as the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) have historically provided the foundation for inventory decision-making. The EOQ model was introduced [9], while the EPQ model was later developed [10] to account for gradual replenishment through finite-rate production. Although these models are foundational, they are often too restrictive for modern inventory systems characterized by perishability, shortage risk, and supply chain disruptions.

Inventory is any stored resources that are used to satisfy the present or future needs of an organization. The overall objectives of inventory management are to find the optimal policy of when and how much to produce/order during a given period. An economic production quantity (EPQ) model is an inventory control model that determines the optimal quantity to be produced so as to meet a deterministic demand with an objective of minimizing cost. In general, almost all items on inventory deteriorate over time. Sometimes the rate of deterioration is too low; in reality, not all kinds of items deteriorate as soon as they are produced, but they will maintain their freshness before they begin to deteriorate.

Objectives of Inventory Control

Inventory control has two major objectives. The first objective is to maximize the level of customer service by avoiding understocking. Understocking causes missed deliveries, backlogged orders, lost sales, and unhappy customers. The second objective of inventory control is to promote efficiency in production or purchasing by minimizing the cost of providing a suitable level of customer service. Placing too much emphasis on customer service can lead to higher holding costs, which means the company has tied up too much of its capital in inventories. These two objectives often clash. Achieving high levels of customer service by maintaining certain inventories leads to higher inventory costs and less efficiency in production or purchasing. Inventory control becomes a balancing action. Many a time a manager selects a desired level of customer service and attempts to control inventory in a way that achieves that level of customer service at the lowest cost possible. Thus, the problem is striking a balance in inventory levels, avoiding both overstocking and understocking.

This study develops a two-phase production EPQ model integrating:

1. Non-instantaneous deterioration (items retain freshness before spoilage begins)

2. Stock-dependent demand (customers' demand depends on visible inventory)

3. Partial backlogging (not all unmet demand is backlogged)

4. Environmental costs (green and carbon-aware inventory)

The model determines the optimal production stopping time, cycle length, and shortage duration to minimize total cost while enhancing operational resilience.

Organization of the Paper

This article is organized as follows: Section 1 contains the introduction, objectives, and organization of the study; Section 2 presents the related work in inventory modeling for deteriorating items, demand structures, shortages, and COVID-19 disruptions; Section 3 provides the assumptions, notations, and mathematical formulation of the proposed model; Section 4 describes the theory, calculation, and solution algorithms including optimality conditions; Section 5 presents the numerical results and discussion; Section 6 provides sensitivity analysis; Section 7 concludes the research work with future directions.

Related Work

Inventory management has remained a major area of study in operations research because of its role in balancing cost, service level, and production efficiency. Classical models such as the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) have historically provided the foundation for inventory decision-making. The EOQ model was introduced [9], while the EPQ model was later developed [10] to account for gradual replenishment through finite-rate production. Although these models are foundational, they are often too restrictive for modern inventory systems characterized by perishability, shortage risk, and supply chain disruptions.

Inventory Models for Deteriorating Items

One of the most important developments in inventory theory is the incorporation of deterioration. In many practical systems, particularly those involving food, pharmaceuticals, chemicals, and agricultural products, items lose value or utility over time. [11] Provided one of the earliest and most influential treatments of perishable inventory systems, laying the foundation for subsequent studies on deteriorating items.

However, in many real-life situations, products do not deteriorate immediately after production or replenishment. Instead, they remain usable or fresh for a finite period before spoilage begins. This gave rise to the concept of non-instantaneous or delayed deterioration, which is especially relevant for products such as meat, bread, cassava, dairy products, and medical supplies. More recent works have extended classical deteriorating inventory models to account for this delayed spoilage behaviour [12], [23], [24]. These extensions improve realism by better representing the shelf-life structure of perishable and semi-perishable goods.

Demand Structures and Stock-Dependent Demand

Demand assumptions play a critical role in inventory modeling. Traditional models often assume constant demand, but this assumption may not hold in retail or visible-stock environments where customer purchasing behaviour is influenced by displayed inventory. In such settings, demand may increase when stock levels are high, reflecting consumer confidence, freshness perception, or panic purchasing tendenrrrcies. This phenomenon is known as stock-dependent demand**.** [13] and [12] showed that stock visibility can significantly affect demand patterns and optimal replenishment decisions. Such demand structures are especially relevant for perishable goods, where display stock can directly influence customer buying behavior and depletion speed. This becomes even more important in post-disruption environments where availability itself can stimulate demand.

Shortages and Partial Backlogging

Shortages are another important feature of realistic inventory systems. Classical inventory models frequently assume either no shortages or full backlogging, where all unmet demand is delayed and satisfied later. In practice, however, customer behaviour is more complex. Some customers are willing to wait, while others abandon the purchase when stock-outs occur. To capture this behaviour, partial backlogging models were developed. [14] Showed that customer willingness to wait often decreases with increasing shortage duration. This idea has been extended in later inventory studies, including those involving deterioration and replenishment trade-offs [19], [24]. Partial backlogging is particularly useful in disruption-prone environments where customer patience is uncertain and shortages may be unavoidable.

Production-Oriented Inventory Models and EPQ Extensions

Although EOQ models are suitable for procurement systems, EPQ models are more appropriate for manufacturing environments where replenishment occurs through production over time. As a result, EPQ models have been widely extended to incorporate deterioration, shortages, and dynamic demand structures.

In practice, production is not always carried out in a single uninterrupted phase. Manufacturing systems may operate under capacity limitations, machine breakdowns, labor shortages, maintenance schedules, or external shocks. This has motivated the development of multi-phase production models**,** which more accurately reflect actual production environments. Such structures became particularly relevant during and after the COVID-19 pandemic, when many firms shifted between restricted production and gradual recovery states.

Supply Chain Disruptions and COVID-19 Context

The COVID-19 pandemic exposed the vulnerability of global and local supply chains to severe disruption. It affected production capacity, labour availability, transportation systems, customer demand, and inventory replenishment. As a result, conventional deterministic inventory models proved inadequate for many real-life decision environments. [15] Demonstrated that epidemic outbreaks can create ripple effects across supply chains, where localized disruptions spread through interconnected production and distribution networks. Similarly, [16] emphasized the need for operations research models that explicitly support resilience, flexibility, and recovery planning under disruption conditions. [17] Also highlighted the importance of recovery-based production planning for high-demand items during the COVID-19 crisis. These studies strongly justify the need for inventory models that incorporate production interruptions, recovery phases, shortage behaviour, and adaptive demand structures**,** especially for perishable goods. [25] Develops a sustainable Economic Production Quantity (EPQ) model for the COVID-19 pandemic, incorporating green investments and preservation technology (PT). It finds that investing in both technologies reduces greenhouse gas emissions and increases system profit by 16.1%, while managing partial backordering and demand linked to lockdown levels. [29] A COVID-19 Supply Chain Management Strategy Based on Variable Production under Uncertain Environment Conditions

Sustainability and Environmental Cost in Inventory Models

In recent years, sustainability has become an increasingly important issue in inventory and production planning. Modern supply chains are expected not only to minimize economic cost but also to reduce waste, emissions, energy use, and environmental burden. This is especially relevant for deteriorating items, where spoilage and overstocking can create both financial and environmental losses.

[18] Argued that the COVID-19 pandemic should motivate a transition toward more sustainable production and supply systems. In line with this view, several researchers have incorporated environmental and green cost components into inventory models [20], [21]. These approaches are particularly valuable for perishable inventory systems, where excessive holding and deterioration directly increase material waste and disposal burden.

[26] Considered production inventory model for two levels of production and deteriorating items and shortages. [27] Developed production inventory models for deteriorating items with three levels of production and shortages, where they consider constant demand and deterioration in both during and after production.

[28] An EPQ model for non-instantaneous deteriorating Items with stock-dependent demand rates under a two-phase production rate and shortages

Research Gaps

Despite extensive research, few models simultaneously integrate delayed deterioration, stock-dependent demand, partial backlogging, phased production, and environmental costs. This study addresses these gaps by providing analytical solutions, optimality conditions, numerical examples, and sensitivity analysis. The outcomes of this study are expected to enhance supply chain resilience, improve cost efficiency, and support better preparedness for future disruptions.

Assumptions and Notations

Assumptions

1. Single item, infinite planning horizon, and repetitive cycles.

2. Two-phase production: production during the first phase, stopped during the second phase.

3. Non-instantaneous deterioration: items remain fresh for a maturity period td before spoilage at rate θ.

4. Constant demand D during production; stock-dependent demand $a + bI(t)$ after production.

5. Shortages allowed partial backlogging, with rate decreasing over waiting time.

6. Deteriorated items are removed immediately.

7. Total cost includes production, holding, deterioration, shortage, lost sales, and environmental costs.

8. Deterministic system: all parameters known.

Notations

T	Cycle length
t_1	End of production phase
t_d	Start of deterioration

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t_2 Inventory depletion time

$t_3 = T - t_2$ Shortage period

P Production rate

D Constant demand rate

a, b Stock-dependent demand parameters

θ Deterioration rate

δ Backlogging parameter

Q_1 Inventory level at time t_d

Q_2 Inventory level at time t_1

$I(t)$ Inventory level at time t

Imax Maximum inventory

Smax Maximum shortage

Cp, Ch, CD, CS, CL, and CE costs per unit/time for production, holding, deterioration, shortage, lost sales, and environmental

Mathematical formulation of the model

The production cycle begins at t=0 with zero inventory. In the interval $[[0, t_d]]$, inventory grows linearly as the production rate exceeds constant demand (P>D). At t_d, the delayed deterioration begins, resulting in concave growth until the end of the first production phase at t_1. The second production phase occurs during $[[t_1, t_2]]$, reaching the maximum inventory level I_{max}. At t_2, production ceases. During the interval $[t_2, t_4]$, the inventory is depleted by a combination of stock-dependent demand and deterioration. Finally, at t_4, a shortage period begins; the shaded region illustrates the partial backlogging of demand until the end of the cycle T. The process is repeated, and the behavior of the inventory model for one cycle is shown in Figure 2.

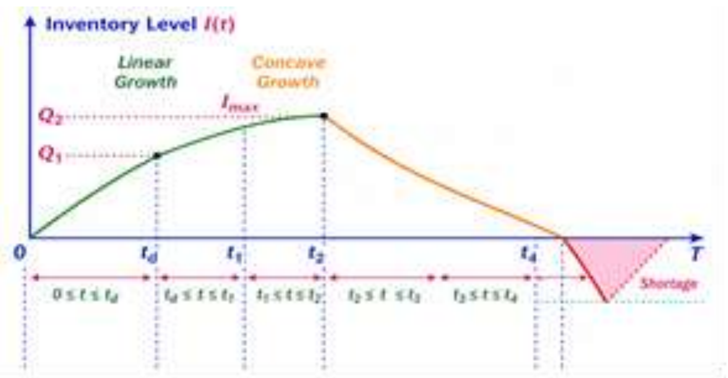

Figure 2. The production inventory system

The differential equations describing the inventory system are given by the equations (1)-(4).

Production without deterioration $(0 \leq t \leq t_1)$

$$\frac{dI(t)}{dt} = P - D, \quad (1)$$

with conditions $I(0) = 0$.

$I_1(t) = (P - D)t$

Production with deterioration $t_d \le t \le t_1$

$\frac{dI(t)}{dt} = P - D - \theta I(t), \quad I(t_d) = I_1(t_d)$ (2)

$$I_2(t) = \frac{P-D}{\theta} + \left(I_1(t_d) - \frac{P-D}{\theta}\right) e^{-\theta(t-t_d)}$$

No production with deterioration $t_1 \le t \le t_2$

$\frac{dI(t)}{dt} = -a - (b+\theta)\, I(t), \quad I(t_2) = 0$ (3)

$I_3(t) = \frac{a}{b+\theta}\left[e^{(b+\theta)(t_2-t)} - 1\right]$

Shortage with partial backlogging $t_2 \le t \le T$

$\frac{dI(t)}{dt} = -\frac{a}{1+\delta(T-t)}, \quad I(t_2) = 0$ (4)

$$I_4(t) = \frac{a}{\delta}\left[\ln\left(1+\delta(T-t)\right) - \ln\left(1+\delta(T-t_2)\right)\right]$$

$S_{max} = -I_4(T) = \frac{a}{\delta}\ln\left(1+\delta(T-t_2)\right)$ (5)

Total Cost Function

$TC(T) = \frac{PC+HC+DC+SC+LSC+EC}{T}$

$PC = C_P P t_1$ (6)

$$HC = C_h\left(\int_0^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt + \int_{t_1}^{t_2} I_3(t)dt\right)$$

$HC = C_h\left[\frac{(P-D)t_d^2}{2} + Q_1(t_1 - t_d) + \frac{(P-D)(t_1-t_d)^2}{2} + \frac{Q_2}{\theta}\left(1 - e^{-\theta(t_2-t_1)}\right)\right]$
(7)

$$DC = C_d\left(\int_{t_d}^{t_1} \theta I_2(t)dt + \int_{t_1}^{t_2} \theta I_3(t)dt\right)$$

$DC = C_d\left(\theta\left[Q_1(t_1 - t_d) + \frac{(P-D)(t_1-t_d)^2}{2}\right] + Q_2\left(1 - e^{-\theta(t_2-t_1)}\right)\right)$ (8)

$$SC = \frac{(P-D)(t_1-t_d)^2}{2}$$

$$SC = C_s\int_{t_2}^{T} -I_4(t)dt$$

$$= C_s\int_{t_2}^{T} a(t-t_2)dt$$

$SC = \frac{C_S a(T-t_2)^2}{2}$ (9)

$$LSC = C_l \int_{t_2}^{T} a\left[1 - \frac{1}{1+\delta(T-t)}\right] dt$$

$= C_l a[(T-t_2) - \frac{1}{\delta}\ln\left(1+\delta(T-t)\right)]$ (10)

$EC = C_e(HC + DC)$ (11)

Total cost per cycle

The average total variable cost per unit time is given by

$$TC(T, t_1, t_3) = \frac{1}{T}[PC(t_1) + HC(t_1, t_2) + DC(t_1, t_2) + SC(t_3) + LSC(t_3) + ES(t_1, t_2)]$$

Where:

$T =$ Cycle length

$t_1 =$Production stopping time

$t_3 = T - t_2 =$Shortage period

The optimization problem is to $\min\limits_{T, t_1, t_3} TC(T, t_1, t_3)$

Subject to $0 < t_d < t_1 < t_2 < T,\ T > 0$

Theory / Calculation, Solution Algorithms

The necessary optimality conditions

Lemma 1: If (T^*, t_1^*, t_3^*) are interior optimal solutions, then they must satisfy

$\frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial t_3} = 0 \quad and \quad \frac{\partial TC}{\partial T} = 0$ (12)

Recall, $TC = \frac{K(T, t_1 t_3)}{T}$ (13)

Where $K(.)$ is the total cost per cycle

$\frac{\partial TC}{\partial T} = \frac{T\frac{\partial K}{\partial T} - k}{T^2}$ (14)

Setting this to zero, we have $T\frac{\partial K}{\partial T} - k = 0$

This implies $\frac{\partial K}{\partial T} = \frac{K}{T}$ (15)

Where

$K = PC(t_1) + HC(t_1, t_2) + DC(t_1, t_2) + SC(t_3) + LSC(t_3) + ES(t_1, t_2)$
(16)

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Now, $$\frac{\partial TC}{\partial t_1} = \frac{1}{T}\left[C_p P + C_h I(t_1) + C_d \theta I(t_1) + C_e(C_h I(t_1) + C_d \theta I(t_1))\right]$$ (17)

Setting this to zero,

$$C_p P + (C_h + C_d\theta)(1 + C_e)I(t_1) = 0 \qquad (18)$$

Where $PC = C_p P t_1$, $0 < t < t_1$ and $t_d < t < t_1$

Recall $S_{max} = \frac{a}{\delta}\ln\left(1 + \delta(T - t_2)\right)$

Thus,

$$\frac{\partial TC}{\partial t_3} = \frac{1}{T}\left[C_s \frac{a}{1+\delta t_3} + aC_l\left(1 - \frac{1}{1+\delta t_3}\right)\right] \qquad (19)$$

Setting this to zero,

we have, $\frac{a}{1+\delta t_3}(C_s - C_l) + aC_l = 0$ (20)

Lemma 2: If $(C_s - C_l) > 0$, then the stationary point (T^*, t_1^*, t_3^*) is a strictly local minimum if the Hessian matrix H is positive definite at that point.

The sufficient conditions for the existence of minima are

$$\frac{\partial^2 TC}{\partial T^2} = \frac{2K}{T^3} > 0$$

$$\frac{\partial^2 TC}{\partial t_1^2} = \frac{1}{T}(C_h + C_d\theta)(1 + C_e)\frac{dI(t_1)}{dt_1} > 0$$

For $\frac{dI(t_1)}{dt_1} > 0$

$$\frac{\partial^2 TC}{\partial t_3^2} = \frac{a\delta}{T(1 + \delta t_3)^2}(C_s - C_l) > 0$$

$$\frac{\partial^2 TC}{\partial T \partial t_3} = \frac{\partial^2 TC}{\partial t_1 \partial t_3} = \frac{\partial^2 TC}{\partial T \partial t_1} > 0$$

Since cross-partial derivatives are finite and symmetric

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T \partial t_3} \\ \frac{\partial^2 TC}{\partial t_1 \partial T} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial t_3} \\ \frac{\partial^2 TC}{\partial t_3 \partial T} & \frac{\partial^2 TC}{\partial t_3 \partial t_1} & \frac{\partial^2 TC}{\partial t_3^2} \end{bmatrix} > 0$$

All the principal minors of H are positive,

Since $\frac{\partial^2 TC}{\partial T^2} > 0$, $\begin{vmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial t_1} \\ \frac{\partial^2 TC}{\partial t_1 \partial T} & \frac{\partial^2 TC}{\partial t_1^2} \end{vmatrix} > 0$ and $|H| > 0$

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Hence, H is positive definite

Theorem 1: $TC(T, t_1, t_3)$ exist, unique and convex

Proof A unique optimal solution (T^*, t_1^*, t_3^*)exists from lemma 1 above. The solution satisfied the first-order necessary condition and second order sufficient conditions. From lemma 2., the total average cost function $TC(T, t_1, t_3)$is strictly convex in $TC(T, t_1, t_3)$

Optimization Algorithm (Newton–Raphson Method)

To determine the optimal values of multiple decision variables simultaneously, the multivariable Newton–Raphson algorithm is employed. Let the decision variable vector be

$X = \begin{bmatrix} T \\ t_1 \\ t_3 \end{bmatrix}$ where $T, t_1, and\ t_3$ denote the cycle time, production switching time, and shortage period, respectively.

The objective is to minimize the total cost function TC (T, t_1, t_3). At the optimum point, the gradient of the total cost function is zero, that is $\nabla TC(X) = 0$

Since the resulting system of non-linear equations generally cannot be solved analytically, the multivariable Newton-Raphson iterative method is used. The formula is given by

$X^{k+1} = X^k - H^{-1}(X^k)\, \nabla TC(X^k)$

Where; X^k is the current estimate of the decision variable vector.

X^{k+1} is the updated estimate

$\nabla TC(X^k) =$ is the gradient vector evaluated at the current iteration,

$H(X^k) =$ is the Hessian matrix of second-order partial derivatives

The gradient vector is defined as $\nabla TC(X) = \begin{bmatrix} \frac{\partial TC}{\partial T} \\ \frac{\partial TC}{\partial t_1} \\ \frac{\partial TC}{\partial t_3} \end{bmatrix}$

The corresponding Hessian matrix is

$$H(X) = \begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T \partial t_3} \\ \frac{\partial^2 TC}{\partial t_1 \partial T} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial t_3} \\ \frac{\partial^2 TC}{\partial t_3 \partial T} & \frac{\partial^2 TC}{\partial t_3 \partial t_1} & \frac{\partial^2 TC}{\partial t_3^2} \end{bmatrix}$$

The iterative process is repeated until convergence is achieved, typically when the norm of the vector satisfies $\|\nabla TC(X^k)\| < \varepsilon$, where ε is the tolerance, which is 10^{-6} . At convergence, the Hessian matrix should be positive definite to ensure that the obtained solution corresponds to a local minimum of the total cost function.

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This multivariable optimization procedure provides an efficient numerical approach for obtaining the optimal values of the decision variables (T, t_1, t_3) particularly when the total cost function is highly nonlinear and analytical solutions are not feasible.

Input: Initial guesses T_0, t_{1_0}, t_{3_0}, and tolerance epsilon

Output: Optimal variables T^*, t_1^*, t_3^*

Step 1: Set iteration counter n = 0.

Step 2: LOOP

Step 3: Calculate the numerical values of the partial derivatives (gradient vector).

Step 4: Calculate the numerical values of the second-order partial derivatives (Hessian Matrix)

Step 5: Compute the inverse of the Hessian matrix: H^{-1}

Step 6: Update the variables: re-explain

$$X_{n+1} = X_n - H^{-1}\nabla TC(X)$$

Step 7: IF $\|X_{n+1} - X_n\| < \varepsilon$ THEN

Stop and output X_{n+1} as the optimal solution T, t_1, t_3

ELSE

Set n = n + 1

Repeat from Step 3

RESULTS AND DISCUSSION

The following discuss the numerical solution, sensitivity analysis, convexity, and managerial implications of the proposed model:

Numerical example

Let us consider the cost parameters $p = 500$, $D = 300$, $a = 200$, $b = 0.4$, $\theta = 0.15$,$\delta = 1.2$, $C_p = 20$, $C_h = 4$, $C_d = 6$, $C_s = 25$, $C_l = 8$, and $C_e = 0.3$ Optimal solution: $T^* = 1.64$, $t_1^* = 0.71$, $t_3^* = 0.13$, $I_{max} \approx 141$, $S_{max} \approx 23$ and $TC(T^*) \approx 12180$

Table 1. Cost breakdown

Component	**Cost Value**	**Cost (%)**
Production	7100	58.3%

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Holding	1810	14.9%
Deterioration	815	6.7%
Shortage	1985	16.3%
Lost Sales	115	0.9%
Environmental	355	2.9%
Total Cost	₦ 12180	100%

Based on the mathematical model and the provided numerical example, this figure 2, illustrates the Total Variable Cost (TC) as a function of the Cycle Length T

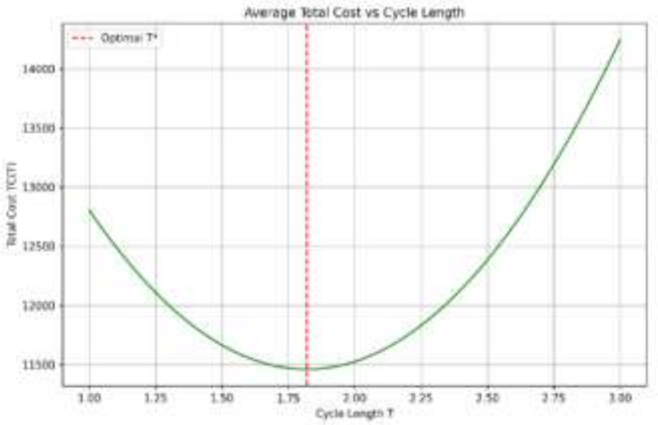

Figure 3. The figure shows the average total cost versus cycle length.

Convexity and Global Minimum

Convex Nature: The green curve exhibits a distinct parabolic shape, confirming that the total cost function is strictly convex with respect to the cycle length T.

Optimal Point: The red dashed vertical line marks the optimal cycle length, calculated in the numerical example as $T^* = 1.64$ units.

Minimized Cost: At this optima T^*, the average total variable cost reaches its lowest point, approximately 12180.

Sensitivity analysis

Table 2. Sensitivity analysis

Percentage Change in Total Cost at ±10% and ±20% Parameter Deviations

Parameter	**-20%**	**-10%**	**+10%**	**+20%**
Deterioration Rate (θ)	-3.85	-1.70	+1.82	+3.90
Shortage Cost (C_s)	-3.26	-1.63	+1.63	+3.26

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Stock-Dependent Demand (b)	-1.90	-0.92	+1.02	+2.15
Lost Sales Cost (C_l)	-0.18	-0.09	+0.09	+0.178
Demand Rate (D)	-13.20	-7.10	+8.40	+18.95
Production cost (p)	+5.06	+1.75	-1.31	-2.40

Sensitivity analysis is essential for evaluating the robustness of the proposed model and understanding how changes in system parameters affect optimal decisions and total cost.

Effect of Deterioration Rate θ

As the deterioration rate increases:

i. The total average cost increases.

ii. the maximum inventory level decreases,

iii. and the system tends to favor shorter inventory cycles.

This is expected because faster deterioration leads to higher spoilage and environmental burden.

Effect of Backlogging Parameter δ

As the backlogging parameter increases:

i. fewer customers are willing to wait,

ii. Lost sales cost rises.

iii. And the shortage period must be controlled more carefully.

This indicates that customer impatience significantly affects optimal shortage policy.

Effect of Stock-Dependent Demand Parameter b

An increase in b implies that demand becomes more responsive to visible stock levels. As a result:

i. inventory depletes faster,

ii. holding cost may reduce,

iii. But production planning becomes more sensitive to display-stock decisions.

Effect of Environmental Cost Coefficient Ce

An increase in environmental cost causes:

i. greater penalty for excessive holding and deterioration,

ii. shorter and more efficient cycles,

iii. and a stronger preference for reduced spoilage.

This confirms that environmental considerations can significantly influence optimal production and inventory policy.

Managerial Implications

The proposed model provides several useful insights for managers and manufacturers dealing with perishable and semi-perishable products:

i. Post-disruption production planning: Firms can use the model to design recovery-oriented production schedules after disruptions such as pandemics, lockdowns, or raw material shortages.

ii. Spoilage control: Delayed deterioration modeling helps managers reduce unnecessary overproduction and waste.

iii. Customer-service strategy: Partial backlogging allows firms to better understand the cost trade-off between customer waiting and lost sales.

iv. Demand-responsive inventory display: Stock-dependent demand implies that inventory visibility can be strategically managed to stimulate sales without causing excessive spoilage.

v. Green operations: Incorporating environmental costs encourages more sustainable inventory decisions and supports cleaner production planning.

CONCLUSION AND FUTURE SCOPE

This study proposed an economic production quantity model that integrates two-phase production, non-instantaneous deterioration, stock-dependent demand, partial backlogging, and environmental costs. The model addresses critical gaps in the inventory management literature by providing a framework specifically designed for post-pandemic supply chain recovery. The analytical solutions demonstrate that careful production scheduling and controlled shortages reduce total cost, limit deterioration, and improve sustainability. The necessary and sufficient optimality conditions were established, proving the existence and uniqueness of the optimal solution. The Newton-Raphson method was applied for numerical optimization, and convexity was verified both analytically and graphically. This study contributes to the inventory and production planning literature by introducing explicit COVID-19 disruption considerations into a two-phase EPQ framework, integrating delayed deterioration with stock-dependent demand and partial backlogging under pandemic conditions, providing comparative insights between disrupted and normal production systems, offering a mathematically rigorous yet practically applicable model for post-pandemic manufacturing recovery, and incorporating environmental cost into the inventory decision structure.

The model is practical for perishable supply chains dealing with products such as meat, bread, cassava, dairy, and pharmaceutical items. Manufacturers can apply this framework to determine optimal production stopping times, cycle lengths, and shortage durations that minimize total cost while enhancing operational resilience. The inclusion of environmental costs aligns inventory decisions with sustainability goals, which is increasingly important for regulatory compliance and corporate social responsibility. Future research may extend the current study in several directions. These include the incorporation of stochastic demand rather than deterministic assumptions, extension to multi-item systems with interdependent products, inclusion of inflation and time value of money for long-term planning, consideration of imperfect production and rework processes, development of fuzzy or probabilistic versions of the model to handle uncertainty, and integration with real-time data and adaptive control systems for dynamic inventory management.

Study limitations

None

Conflict of interest

All authors declare that they do not have any conflict of interest.

Authors' Contributions

All authors reviewed and edited the manuscript and approved the final version of the manuscript.

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Data Availability

The study presents a theoretical inventory model based on differential equations, supported by numerical simulations and sensitivity analysis to illustrate system behavior and provide indicative managerial insights within the scope of the model assumptions.

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