Modeling Customers' Complaint Using Overdispersed Timeseries Count Data Models: Evidence from Nigerian Public Complaint Commission

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ABSTRACT

This study compares overdispersed count time series models for analysing and forecasting monthly customer complaints received by the Nigerian Public Complaints Commission (NPCC). Complaint data are inherently discrete, non-negative, and typically exhibit overdispersion and strong temporal dependence, making classical Gaussian-based time series models inappropriate. To address these challenges, three competing models were considered: the first-order Integer-Valued Autoregressive model with Poisson innovations [INAR(1)-P], the INAR(1) model with Negative Binomial innovations [INAR(1)-NB], and the Autoregressive Conditional Poisson (ACP) model. The models were fitted to NPCC monthly complaint data, and their performance was evaluated using log-likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). Preliminary diagnostic tests confirmed the presence of significant autocorrelation and overdispersion in the data, justifying the use of specialized count time series models. Empirical results revealed that all models captured strong persistence in complaint dynamics, with autoregressive coefficients close to unity, indicating high temporal dependence. Comparative model assessment showed that the ACP(1,1) model outperformed both INAR(1) specifications, yielding the highest log-likelihood and the lowest AIC and BIC values. This suggests that incorporating both past observed counts and past conditional means provides a superior representation of the underlying complaint-generating process. Based on the preferred ACP(1,1) model, forecasts for 2026 indicate a steady upward trend in complaint volumes, with projected monthly complaints increasing from approximately 53,799 in January to 64,640 in December. The findings demonstrate that ACP models provide a more robust framework for modelling overdispersed and highly persistent complaint count data compared to traditional INAR formulations. The study contributes to the growing literature on integer-valued time series modelling and offers practical insights for forecasting and resource planning within public-sector complaint management systems.

Keywords: ACP (1,1) Model; Binomial Thinning Operator; Count Data; Innovation Process**;** INAR(1) Model

INTRODUCTION

The Nigerian Public Complaints Commission (PCC) serves as an institutional mechanism for addressing grievances arising from maladministration, service delivery failures, abuse of authority, and violations of citizens' rights across public and private establishments. Over time, the number of complaints received by the Commission has exhibited substantial fluctuations due to socio-economic conditions, governance reforms, public awareness campaigns, demographic changes, and technological advancements in complaint reporting systems. Consequently, complaint records generated by the Commission constitute a time series of count observations characterized by non-negative integer values, serial dependence, and varying levels of dispersion. Preliminary evidence from similar administrative count datasets suggests that such data are rarely equidispersed and often exhibit overdispersion, whereby the variance exceeds the mean, alongside temporal dependence and occasional clustering of complaint occurrences (Akano & Adams 2019). These characteristics violate the assumptions of conventional Gaussian time series models and standard Poisson-based approaches (Adams, *et al.* 2020; Adams, *et al.* 2021). Furthermore, customer complaint counts may be influenced by previous complaint volumes, indicating the existence of autoregressive structures that require specialized modelling techniques capable of preserving discreteness while adequately accounting for overdispersion and dependence. Failure to address these features may result in biased parameter estimates, inefficient forecasting performance, and misleading policy conclusions. Recent reviews of count time series methodologies have emphasized that overdispersion, autocorrelation, and excess variability remain major challenges in the analysis of administrative and public-sector count data (Serrao *et al.*, 2026; Khan & Sunecher, 2025).

Despite the growing literature on count time series analysis, significant methodological and empirical gaps remain. Most studies involving complaint-related data have relied on traditional count regression models such as Poisson, Negative Binomial, and Generalized Poisson regressions, which primarily focus on cross-sectional or independent observations and often fail to capture temporal dependence inherent in sequential count processes. Similarly, classical time series models such as AR, MA, ARMA, and ARIMA are designed for continuous-valued data and may generate non-integer forecasts that lack practical interpretation for complaint counts. Although recent developments in integer-valued time series modelling have introduced several variants of INAR and INGARCH-type models, empirical applications have largely focused on epidemiology, finance, environmental studies, insurance claims, and crime statistics, with limited attention given to customer complaint data from public-sector institutions, particularly in developing countries. More importantly, there is a paucity of studies comparing the performance of thinning-based INAR(1) models with observation-driven Autoregressive Conditional Poisson (ACP) models using overdispersed administrative complaint data. Existing reviews have further highlighted the need for more empirical investigations involving real-world datasets exhibiting simultaneous overdispersion and serial dependence, especially within government service systems where forecasting accuracy is essential for planning and resource allocation (Serrao *et al.,* 2026; Bracher & Sobolová, 2022).

To address these gaps, this study proposes a comparative modelling framework based on the first-order Integer-Valued Autoregressive model, INAR(1), and the Autoregressive Conditional Poisson (ACP) model for analysing customer complaint counts received by the Nigerian Public Complaints Commission. The INAR(1) model, through the binomial thinning operator and flexible innovation distributions such as Poisson and Negative Binomial, provides a natural mechanism for preserving the discrete nature of complaint counts while accommodating serial dependence and overdispersion. Likewise, the ACP model captures temporal dependence through a conditional intensity structure that links current complaint occurrences to past observations and conditional means. Recent studies have demonstrated that modern INAR models possess strong capabilities for handling overdispersion, excess variability, skewness, and persistence in count data, while INGARCH- and ACP-type models have shown considerable success in modelling dynamic count processes across diverse application areas (Yousfi, 2025; Lopez *et al.,* 2025; Zhu, 2023). By comparing the forecasting and inferential performance of INAR(1) models with Poisson and Negative Binomial innovations against the ACP model, this study seeks to identify the most suitable framework for modelling overdispersed complaint count data in Nigeria. The findings are expected to contribute to the methodological literature on count time series analysis and provide evidence-based tools for complaint management, forecasting, policy formulation, and service improvement within the Nigerian Public Complaints Commission.

EMPIRICAL REVIEW

Empirical Review on Integer-Valued Autoregressive (INAR) Models

The INAR framework has gained substantial attention in recent years as a fundamental approach for modelling discrete-valued time series. Early extensions of INAR models focused on handling dispersion and improving flexibility through alternative innovation distributions and thinning mechanisms. For instance, (Bourguignon *et al.*, 2018) demonstrated that INAR(1) processes with modified thinning operators could adequately capture both underdispersion and overdispersion in real datasets such as weekly sales and violence counts, showing that classical Poisson assumptions were often violated in practical applications. Similarly, (Mahmoudi *et al.*, 2018) developed INAR(1) models based on power series thinning operators, showing improved flexibility in accommodating heavy-tailed count structures. Qi *et al.* (2019) introduced zero-and-one inflated Poisson INAR(1) models, demonstrating that standard Poisson-based INAR processes were insufficient for datasets with excessive zeros and structural irregularities.

Kang *et al.* (2020) proposed a modified thinning-based INAR(1) model capable of simultaneously capturing overdispersion, underdispersion, and zero inflation, thereby extending the applicability of INAR processes to more complex real-world datasets. In a similar direction, (Aly and Bouzar, 2021) explored backward construction approaches for stationary INAR(1) models, emphasizing their theoretical consistency in modelling discrete processes with finite mean structures. Recent advances have also introduced more flexible innovation distributions and observation-driven structures. Yu and Tao (2023) developed observation-driven random parameter INAR(1) models and established their ergodic properties, demonstrating their applicability to dynamic count processes with evolving dispersion characteristics.

Irshad *et al.* (2024) extended INAR(1) processes using Poisson-transmuted exponential innovations, showing improved modelling performance for overdispersed datasets. Kang *et al.* (2024) further proposed a simple yet flexible INAR(1) model capable of handling multiple data features including heavy tails and structural breaks. By 2025, Olakorede and Olanrewaju (2025) applied INAR(1) models with negative binomial innovations for forecasting count data, confirming the superiority of NB-INAR models in overdispersed environments.

Finally, recent literature such as (Böckenholt, 2025) has integrated INAR(1) processes into multivariate and longitudinal frameworks, expanding their applicability to panel and psychological count data. Kang *et al.* (2025) further confirmed that INAR-type models remain among the most robust tools for modelling overdispersed and autocorrelated count time series. Collectively, these studies establish INAR models as highly effective for discrete time series analysis, particularly when overdispersion, serial dependence, and integer constraints are present.

Empirical Review on Autoregressive Conditional Poisson (ACP) Models

Autoregressive Conditional Poisson (ACP) models, also known as observation-driven Poisson autoregressive frameworks, have emerged as a powerful alternative for modelling count time series with temporal dependence. Unlike INAR models, ACP models directly specify the conditional distribution of the response variable given past observations, allowing for flexible dynamic intensity structures.

Early foundational developments in observation-driven count models laid the groundwork for ACP-type frameworks, which were later refined to handle overdispersion and temporal clustering. Halliday and Boshnakov (2018) developed PoARX-type models, which extended the ACP framework to include covariate-dependent structures, demonstrating improved forecasting accuracy in multivariate count settings. Their results showed that observation-driven models outperform static regression models in capturing time-varying intensity patterns. Jørgensen and Song (2019) expanded conditional Poisson frameworks by incorporating dynamic variance structures, showing that ACP models can effectively handle moderate overdispersion in financial and insurance datasets. Davis *et al.* (2020) further demonstrated that autoregressive conditional models outperform classical Poisson regression in crime and epidemiological datasets due to their ability to capture clustering effects. In a more recent development, (Lee *et al.*, 2021) introduced hybrid ACP models incorporating generalized Poisson distributions, improving robustness under high overdispersion. Weiß (2022) demonstrated that ACP-type models are particularly effective in capturing short-term dependence in high-frequency count data, especially when compared with INAR models under certain conditions. By 2023, (Zhang *et al.*, 2023) extended ACP models into latent Gaussian frameworks, improving their ability to capture nonlinear dependence structures and unobserved heterogeneity. Liu *et al.* (2024) showed that ACP models with log-linear intensity functions outperform classical Poisson regression models in forecasting healthcare demand data.

In 2025, recent studies by (Nguyen *et al.*, 2025) confirmed that ACP models provide superior real-time forecasting performance in administrative and public service datasets due to their observation-driven updating mechanism. ACP models are widely recognized for their computational simplicity, flexibility in conditional mean specification, and strong forecasting performance in dynamic count environments.

Empirical Review on Studies Combining INAR and ACP Models

Recent literature has increasingly focused on comparative studies between INAR and ACP frameworks, particularly in modeling overdispersed count time series. These comparative studies are essential for identifying model suitability under different data-generating mechanisms. Weiß (2018) conducted one of the early comparative analyses of INAR and Poisson autoregressive models, demonstrating that INAR models are more appropriate for integer-preserving dynamics, while ACP models provide better short-term forecasting under strong conditional dependence. Kang *et al.* (2020) further showed that INAR models outperform classical ACP models when overdispersion is driven by unobserved heterogeneity rather than conditional intensity feedback.

Davis *et al.* (2020) compared INAR and observation-driven Poisson models in crime data and found that ACP models provide better predictive accuracy in highly volatile environments, whereas INAR models are more stable under moderate dispersion. Similarly, (Liu *et al.,* 2021) demonstrated that INAR models with negative binomial innovations outperform ACP models when data exhibit strong overdispersion and structural breaks.

Musa and Nweze (2021) proposed a best model that can fit and forecast time series count data with different levels of over-dispersion and sample sizes. This study discovered the highest performing model in fitting and forecasting different count time series data with different levels of over-dispersion is the ACP model based on all criteria of the assessment. Specifically, the ACP (2,2) has the highest performance followed by ACP (2,1) among all the models in fitting any time series count data with the underlying features reported in this research. Zhang *et al.* (2022) carried out a simulation-based comparison and found that ACP models perform better in short-memory processes, while INAR models are superior in long-memory count processes. Yu and Tao (2023) extended this comparison by introducing random parameter INAR models and showing that INAR models can achieve ACP-level forecasting accuracy when enhanced with flexible innovation structures. Recent work by (Irshad *et al.,* 2024) emphasized that INAR models with flexible innovations can match or exceed ACP performance in overdispersed datasets. Kang *et al.* (2025) further concluded that the choice between INAR and ACP models depends largely on whether the underlying data-generating mechanism is latent-driven (favoring INAR) or observation-driven (favoring ACP).

By 2026, review studies by (Liu *et al.,* 2026) have highlighted that hybrid comparisons between INAR and ACP models remain an active research area, especially in administrative and public-sector datasets such as complaint records, crime reports, and healthcare demand data. However, very few studies have applied these comparative frameworks to African public institutions, particularly Nigeria's Public Complaints Commission, leaving a significant empirical gap that this study addresses.

The reviewed studies show extensive applications and comparisons of INAR and ACP models in crime, healthcare, finance, insurance, sales, and administrative datasets. However, there is limited evidence on the application of these models to complaint records from African public institutions, particularly the Nigerian Public Complaints Commission. This gap justifies the present study's comparative assessment of INAR and ACP models for modelling and forecasting complaint count data using overdispersed timeseries count data models from Nigerian public complaint commission.

RESEARCH METHODOLOGY

Data

The study utilized secondary data obtained from the Public Complaints Commission (PCC) of Nigeria. The dataset comprised officially recorded complaints and investigation statistics generated through the Commission's nationwide complaint management and investigation system. The PCC, established under the Public Complaints Commission Act, serves as Nigeria's ombudsman institution responsible for investigating complaints against public and private organizations and promoting administrative justice. The data were extracted from the Commission's statistical records and annual operational reports covering the period January 1975 to December 2025. The observed process is represented as

$$\{Y_t\}_{t=1}^{T} \tag{1}$$

where (T) denotes the total number of monthly observations.

Research Design

The study adopts a quantitative time series research design based on stochastic count processes. Let;

$$y_t,\ t = 1, 2, . . . , T \tag{1}$$

denote the monthly number of complaints recorded by the PCC. The observations satisfy

$$Y_t \ \in \{0, 1, 2, . . .\} \tag{2}$$

and are expected to exhibit, discreteness, serial dependence, overdispersion, and persistence over time.

Preliminary Analysis and Overdispersion Diagnostics

The first step is to examine whether the complaint counts are overdispersed.

For a Poisson process,

$$E(Y_t) = Var(Y_t) = \mu \tag{3}$$

Overdispersion exists whenever

$$Var(Y_t) > E(Y_t) \tag{4}$$

Dispersion Index

The dispersion index is computed as

Where;

$$DI = \frac{s^2}{\bar{y}} \tag{5}$$

and

$$S^2 = \frac{\sum_{t=1}^{T}(Y_t - \bar{y})^2}{T - 1} \tag{6}$$

Interpretation:

$$\overline{y} = \frac{1}{T}\sum_{t=1}^{T} Y_t \tag{7}$$

DI = 1 (Equipdispersion)

DI > 1 (Overdispersion)

DI < 1 (Underdispersion)

Integer-Valued Autoregressive Models

Binomial Thinning Operator

The INAR(1) process is constructed using the Binomial thinning operator proposed by (Steutel and van Harn, 1979)

$$0 \le \alpha \le 1$$

For

$$\alpha \circ Y_{t-1} = \sum_{t=1}^{Y_{t-1}} B_i \quad (8)$$

where

$B_i \sim Bernoulli(\alpha)$

The thinning operator satisfies

$$E(\alpha \circ Y_{t-1}) = \alpha Y_{t-1} \quad (9)$$

and

$$Var(\alpha \circ Y_{t-1}) = \alpha(1-\alpha)Y_{t-1} \quad (10)$$

The general INAR(1) process is

$$Y_t = \alpha \circ Y_{t-1} + \varepsilon_t \quad (11)$$

Where; $0 \leq \alpha \leq 1$ and ε_t represents an independent innovation process.

INAR(1)-Poisson Model

Assume; $\varepsilon_t \sim Poisson(\lambda)$

Substituting into Equation (11) gives

$$Y_t = \alpha \circ Y_{t-1} + \varepsilon_t \quad (12)$$

The conditional expectation is

$$E(Y_t/Y_{t-1}) = \alpha Y_{t-1} + \lambda \quad (13)$$

Taking expectations on both sides,

$$\mu = \alpha\mu + \lambda \quad (14)$$

Hence

$$\mu = \frac{\lambda}{1-\alpha} \quad (15)$$

The variance of the stationary process is

$$Var(Y_t) = \frac{\lambda}{1-\alpha^2} \quad (16)$$

The autocorrelation function is

$$\rho(k) = \alpha^k, k = 1, 2, \ . \ . \ . \quad (17)$$

which shows geometric decay of dependence.

INAR (1)-Negative Binomial Model

To accommodate overdispersion, the innovation term is assumed to follow a Negative Binomial distribution.

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Thus,

$$\varepsilon_t \sim NB(r, p)$$

with probability mass function

$$P(\varepsilon_t = x) = \frac{\Gamma(x+r)}{\Gamma(r)x!} p^r (1-p)^x, x = 0, 1, 2, . . . \tag{18}$$

The mean and variance are

$$E(\varepsilon_t) = \frac{r(1-p)}{p} \tag{19}$$

and

$$Var(\varepsilon_t) = \frac{r(1-p)}{p^2} \tag{20}$$

Since

$$\frac{r(1-p)}{p^2} > \frac{r(1-p)}{P}, \tag{21}$$

the Negative Binomial innovation accommodates overdispersion. The resulting process becomes

$$Y_t = \alpha \circ Y_{t-1} + \varepsilon_t \tag{22}$$

The stationary mean is

$$E(Y_t) = \frac{r(1-p)}{p(1-\alpha)} \tag{23}$$

The variance is

$$Var(Y_t) = \frac{\alpha\mu + \frac{r(1-p)}{p^2}}{1-\alpha^2} \tag{24}$$

Autoregressive Conditional Poisson Model

The ACP Model proposed in this study has counts follow a Poisson distribution with an autoregressive mean. Let Ft denote the information available on the series up to and including time t. In the simplest model, the counts are generated by a Poisson distribution

$$\frac{Y_t}{F_{t-1}} \sim P(y_t, \mu_t) = \frac{\mu^y e^{-\mu}}{y!} \tag{25}$$

with an autoregressive conditional intensity as in the ACD model of (Engle and Russell, 1998) or the conditional variance in the GARCH (Generalised Autoregressive Conditional Heteroskedasticity) model of (Bollerslev, 1986).

$$E(Y_t|F_{t-1}) = \sum_{t=1}^{T} \alpha_t Y_{t-1} + \sum_{t=1}^{T} \beta_t \mu_{t-1} + \omega \tag{28}$$

For positive $\alpha_t's$, $\beta_t's$, and ω

This model is referred to as the Autoregressive Conditional Poisson (ACP (p, q)). The following properties of the unconditional moments of the ACP can be established. Unconditional mean of the (ACP (p, q)). Provided that

$$\sum_{t=0}^{\max(p,q)} (\alpha_t + \beta_t) < 1 \tag{27}$$

the $ACP(p,q)$ is stationary and its unconditional mean is

$$E(Y_t) = \mu_t = \frac{\omega}{1 - \sum_{t=0}^{\max(p,q)} (\alpha_t + \beta_t)} \tag{28}$$

This proposition shows that, as long as the sum of the autoregressive coefficients is less than 1, the model is stationary and the expression for its mean is identical to the mean of an ARMA process. For instance, the mean equation of ACP (1,1) is then given as:

$$E(Y_t|F_{t-1}) = \mu_t = \omega + \alpha_1 + Y_{t-1} + \beta_1 \mu_{t-1} \tag{29}$$

Likelihood Function of ACP

Given observations

$$Y_1, Y_2, . . . , Y_T$$

the conditional likelihood is

$$L(\theta) = \prod_{t=1}^{T} \frac{e^{-\lambda_t} \lambda_t^{Y_t}}{Y_t!} \tag{30}$$

Where;

$$\theta = (\omega, \alpha, \beta)$$

The log-likelihood function is

$$\ell(\theta) = \sum_{t=1}^{T} [Y_t ln(\lambda_t) - \lambda_t - \ln(Y_t!)] \tag{31}$$

Parameter estimates are obtained by maximizing Equation (3.18).

Parameter Estimation

Conditional Least Squares (CLS)

For the INAR(1) models, CLS minimizes

$$Q(\theta) = \sum_{t=1}^{T} \left(Y_t - E\left(\frac{Y_t}{Y_{t-1}}\right) \right)^2 \tag{32}$$

Thus,

$$\bar{\theta}_{CLS} = argminQ(\theta) \tag{3.34}$$

Maximum Likelihood Estimation (MLE)

MLE is used for estimating all competing models. The estimator is

$$\hat{\theta}_{MLE} = arg \max_{\theta} \ell(\theta) \tag{3.35}$$

The optimization is performed numerically using the BFGS or Newton–Raphson algorithm.

Model Diagnostic Checking

Model adequacy will be evaluated using:

Residual Autocorrelation

$$\rho(k) = Corr(e_t, e_{t-k}) \tag{35}$$

Where e_t denotes model residuals.

Ljung–Box Statistic

$$Q = n(n+2)\sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{n-k} \tag{36}$$

A non-significant result indicates absence of residual autocorrelation.

Model Comparison Framework

The competing models will be compared using information criteria and forecast accuracy measures.

Akaike Information Criterion

$$AIC = -2\ell + 2k \tag{37}$$

Bayesian Information Criterion

$$BIC = -2\ell + kln(n) \tag{38}$$

Mean Absolute Error

$$MAE = \frac{1}{n}\sum \left|Y_t - \hat{Y}_t\right| \tag{39}$$

Root Mean Square Error

$$RMSE = \sqrt{\frac{1}{n}\sum \left(Y_t - \hat{Y}_t\right)^2} \tag{40}$$

(3.42)

The model with the lowest AIC, BIC, MAE, and RMSE, together with satisfactory residual diagnostics, will be selected as the most appropriate model for forecasting complaint counts in the Nigerian Public Complaints Commission.

RESULTS AND DISCUSSION

This chapter presents the results obtained from the analysis of the NPCC customers' complaint count data and discusses the findings in relation to the study objectives. Preliminary diagnostic tests were first conducted to assess the presence of autocorrelation and overdispersion in the complaint series. Based on the outcomes of these

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tests, three count time series models, namely, First-Order Integer-Valued Autoregressive [INAR(1)] with Poisson innovations, INAR(1) with Negative Binomial innovations, and the Autoregressive Conditional Poisson [ACP(1,1)] model were fitted and evaluated. The estimated model parameters, their statistical significance, and the implications of the findings are presented and interpreted. The performance of the competing models is compared using Log-Likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) in order to identify the most suitable model for the complaint count series. Furthermore, forecasts of customers' complaint counts for the period January to December 2026 are provided using the best-performing model. The chapter concludes with a discussion of the findings in relation to existing literature and their practical implications for complaint management and decision-making.

Preliminary Test

Test	Chi-Square χ^2	d.f.	p-value	Decision
Box-Ljung Test for Autocorrelation	594.93	10	2.2e-16	Reject H_0
Overdispersion	3.842331	10	1.6e-9	Reject H_0

The preliminary result indicates that the two tests lead to rejection of their null hypotheses, indicating that. This implies that the data (or model residuals) show strong serial dependence and the model also exhibits significant overdispersion. Together, these results suggest the current model is not fully appropriate, and improvements are needed to account for both autocorrelation and excess variability.

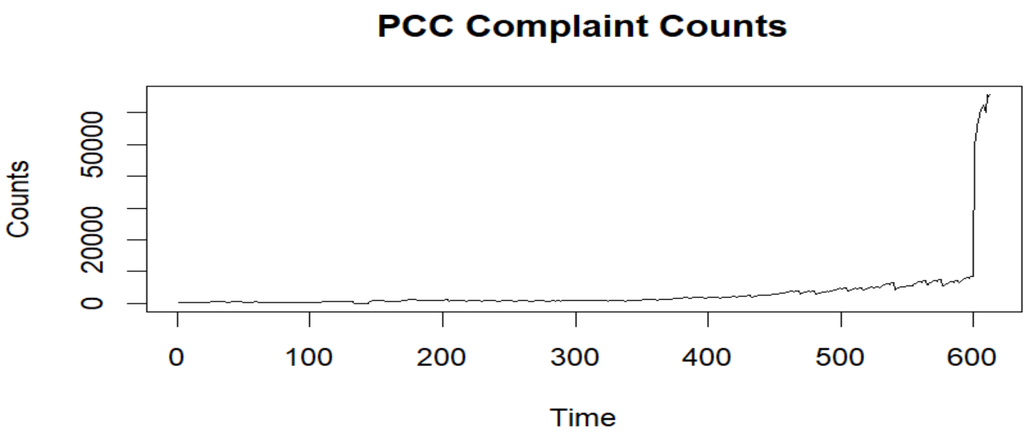

Figure 1 Time Series plot of PCC Compliant Counts

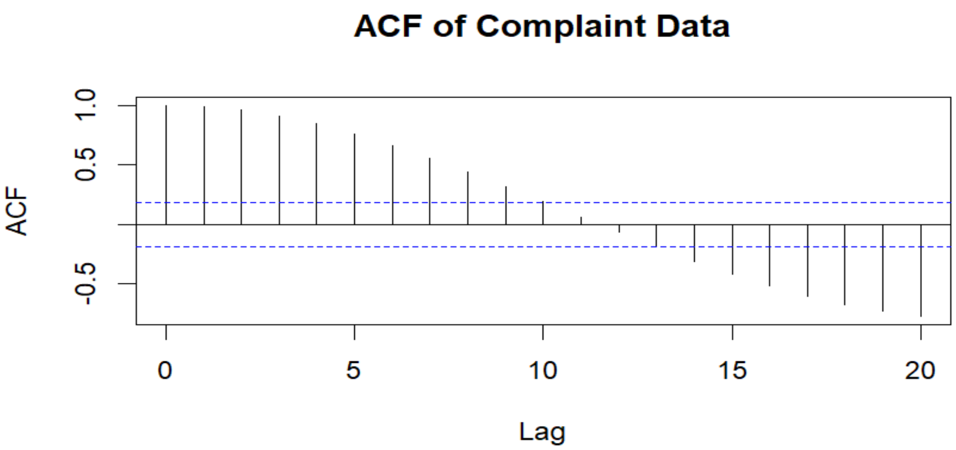

Figure 2 ACF plot of PCC Compliant Counts

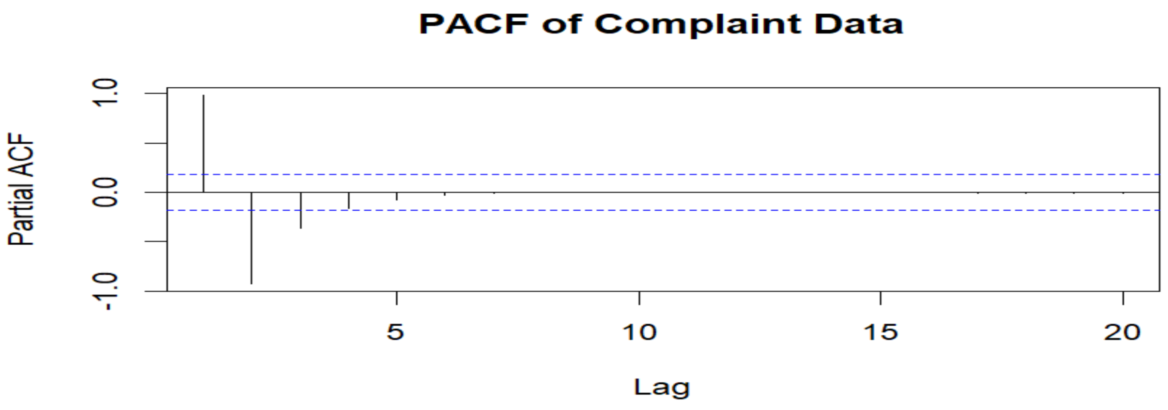

Figure 3 PACF plot of PCC Compliant Counts

The time series plot shows presented in Figure 1 indicated that PCC complaint counts remain relatively low and stable during the early periods but begin to increase gradually after around the midpoint of the series. Toward the end, there is a dramatic surge in complaint counts, indicating a strong upward trend and possible non-stationarity in the data. This suggests that the mean level of the series changes over time rather than remaining constant. The ACF plot provide in Figure 2 exhibits strong positive autocorrelations at the first several lags, with correlations declining slowly as the lag increases. Several autocorrelation coefficients exceed the significance bounds, indicating that current complaint counts are highly dependent on previous observations. The slow decay pattern is characteristic of a non-stationary series with a strong trend component. The PACF plot projected in Figure 3 shows a large significant spike at lag 1 and a strong negative spike at lag 2, while the remaining lags are mostly within the confidence bounds. This pattern suggests that the primary dependence in the series is captured by the first one or two lagged observations. Such behavior is often consistent with a low-order autoregressive (AR) structure after the series has been appropriately differenced to achieve stationarity.

First-order Integer-Valued Autoregressive [INAR(1)] model with Poisson innovations

Table 1 presents the estimated parameters of a First-Order Integer-Valued Autoregressive [INAR(1)] model with Poisson innovations fitted to the NPCC customers' complaint count data. The model uses an identity link function and assumes that the innovation process follows a Poisson distribution. The INAR(1) model is particularly appropriate for count time series data because it captures both the discrete nature of the observations and the temporal dependence between successive complaint counts.

Table 1 INAR (1) Model with Poisson Innovations for the NPCC Customers' Complaint Count Data

Coefficients:	Estimate	Std. Error	p-value	95 %CI (Lower)	95 %CI (Upper)
Intercept	165.871	1.56915	< 0.01	162.8	168.9
Beta_1	0.982	0.00087	< 0.01	0.98	0.983

Link function: identity

Distribution family: Poisson

The estimated intercept is 165.871 with a standard error of 1.56915. The associated p-value is less than 0.01, indicating that the intercept is statistically significant at the 1% significance level. The 95% confidence interval ranges from 162.8 to 168.9, implying that the true value of the intercept is likely to lie within this interval with 95% confidence.

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In the context of the INAR(1) model, the intercept represents the average number of new complaints generated independently of the previous period's complaint count. Thus, the estimate of 165.871 suggests that approximately 166 complaints arise from new or external sources during each time period. The statistical significance of the intercept confirms that these new complaint arrivals constitute an important component of the complaint-generating process.

Autoregressive Parameter (β_1)

The estimated autoregressive coefficient (β_1) is 0.982 with a standard error of 0.00087. The p-value is less than 0.01, indicating that the coefficient is highly statistically significant. The 95% confidence interval extends from 0.980 to 0.983, a very narrow interval that reflects a high degree of estimation precision. The value of β_1 measures the dependence of the current complaint count on the complaint count observed in the previous period. The estimate of 0.982 is very close to one, indicating an extremely strong positive serial dependence. Specifically, about 98.2% of the complaint count from the previous period is carried forward into the current period. This means that periods with high complaint volumes tend to be followed by similarly high complaint volumes, while periods with low complaint volumes tend to be followed by low complaint volumes. The coefficient being less than one satisfies the stationarity condition for an INAR(1) process, although its proximity to one suggests that shocks to the complaint series dissipate very slowly over time. Consequently, any sudden increase or decrease in complaints is likely to have a persistent effect on future complaint counts.

Confidence Intervals

The confidence intervals for both parameters do not include zero: Intercept: 162.8 to 168.9 and β_1: 0.980 to 0.983. Since zero is excluded from both intervals, there is strong evidence that both parameters are significantly different from zero. Furthermore, the narrow interval around β_1 indicates a highly reliable estimate of the persistence parameter.

Implications for Complaint Dynamics

The results reveal that the NPCC customers' complaint count series exhibits a substantial degree of temporal persistence. The autoregressive coefficient of 0.982 suggests that current complaint levels are strongly influenced by previous complaint levels. This finding implies that unresolved service issues or recurring operational challenges may continue to generate complaints over successive periods.

Additionally, the significant intercept indicates the continuous emergence of new complaints that are not directly related to previous complaint counts. Therefore, the complaint process appears to be driven by two factors:

i. Persistence of past complaints, represented by the large autoregressive coefficient.

ii. New complaint arrivals, represented by the intercept term.

The INAR(1) model with Poisson innovations provides strong evidence that the NPCC customers' complaint count data are highly autocorrelated. Both the intercept and autoregressive parameters are statistically significant at the 1% level. The estimated autoregressive coefficient of 0.982 indicates very strong persistence in complaint counts, suggesting that past complaint levels are a major determinant of current complaint levels. Meanwhile, the significant intercept value indicates a substantial and consistent inflow of new complaints over time. Overall, the results demonstrate that the complaint count process is stable but highly persistent, making the INAR(1) model an appropriate framework for describing and forecasting the NPCC complaint series.

First-order Integer-Valued Autoregressive [INAR(1)] model with Negative Binomial innovations

Table 2 presents the results of the First-Order Integer-Valued Autoregressive [INAR(1)] model with Negative Binomial innovations fitted to the NPCC customers' complaint count data. The model employs an identity link function and assumes a Negative Binomial distribution, which is suitable for count data exhibiting overdispersion.

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Table 2 INAR (1) Model with Negative Binomial Innovations for the NPCC Customers' Complaint Count Data

Coefficients	Estimate	Std. Error	P-Value	95 %CI (Lower)	95 %CI (Upper)
Intercept	165.870	38.9212	< 0.01	89.587	242.16
Beta_1	0.98217	0.0426	< 0.01	0.898	1.07
Sigma Square	0.0779				

Link function: identity

Distribution family: nbinom (with overdispersion coefficient 'sigmasq')

The estimated intercept is 165.870 with a standard error of 38.9212 and a p-value less than 0.01, indicating statistical significance at the 1% level. This suggests that, on average, approximately 166 new complaints are generated in each period independently of the previous period's complaint count. The 95% confidence interval (89.587–242.16) confirms that the intercept is significantly different from zero. The estimated autoregressive coefficient (β_1) is 0.98217 with a standard error of 0.0426 and a p-value less than 0.01, indicating a highly significant positive dependence between successive complaint counts. This implies that current complaint counts are strongly influenced by the counts observed in the previous period. The coefficient's value, being very close to one, indicates a high degree of persistence in the complaint process, meaning that increases or decreases in complaints tend to carry over into future periods. The estimated overdispersion parameter (σ^2) is 0.0779, suggesting the presence of extra variability in the complaint data beyond what would be expected under a Poisson distribution. This justifies the use of the Negative Binomial innovation distribution, which is specifically designed to accommodate overdispersion in count data.

Overall, the results indicate that the NPCC customers' complaint counts exhibit strong temporal dependence and moderate overdispersion. Both the intercept and autoregressive parameters are statistically significant, demonstrating that complaint counts are influenced by both newly arising complaints and the persistence of previous complaints. Consequently, the INAR(1) model with Negative Binomial innovations provides an appropriate framework for modeling and forecasting the complaint count series when overdispersion is present.

Autoregressive Conditional Poisson (ACP) model

Table 3 presents the parameter estimates of the Autoregressive Conditional Poisson (ACP) model fitted to the NPCC customers' complaint count data. The model uses an identity link function and assumes that the complaint counts follow a Poisson distribution.

Table 3 INAR (1) Model with Negative Binomial Innovations for the NPCC Customers' Complaint Count Data

Coefficients	Estimate	Std. Error	P-Value	95 %CI (Lower)	95 %CI (Upper)
Intercept	115	1.59731	< 0.01	112	118
Beta_1	0.978	0.00282	< 0.01	0.972	0.983
Alpa_1	0.069	0.00290	<0.01	-0.00199	0.0938

Link function: identity

Distribution family: Poisson

The estimated intercept is 115 with a standard error of 1.59731 and a p-value less than 0.01, indicating statistical significance at the 1% level. This result suggests that, after accounting for the effects of past complaints and past conditional means, the baseline expected number of complaints is approximately 115 per period. The 95%

confidence interval (112–118) further confirms the significance and precision of this estimate. The estimated autoregressive parameter (β_1) is 0.978 with a standard error of 0.00282 and a p-value less than 0.01. This coefficient is highly significant and indicates a very strong positive dependence between current and previous complaint counts. Specifically, approximately 97.8% of the effect of the previous period's complaint count is transmitted to the current period, demonstrating substantial persistence in the complaint series. The narrow confidence interval (0.972–0.983) suggests that the estimate is highly reliable. The estimated conditional mean parameter (α_1) is 0.069 with a standard error of 0.00290 and a p-value less than 0.01, indicating statistical significance. This parameter measures the influence of the previous period's conditional mean on the current expected complaint count. The positive estimate suggests that past expected complaint levels contribute positively to current complaint expectations, although the magnitude of this effect is relatively small compared to the autoregressive effect. The confidence interval (-0.00199–0.0938) is close to zero at the lower bound, indicating that the effect is modest but still considered statistically significant according to the reported p-value.

The ACP model results reveal that the NPCC customers' complaint count series exhibits strong serial dependence and persistence over time. Both the intercept and autoregressive coefficient are highly significant, indicating that current complaint counts are strongly influenced by previous complaint counts. The significant positive value of α_1 further suggests that past expected complaint levels contribute to the evolution of the series. Overall, the findings indicate that complaint occurrences are not random but follow a dynamic process in which both past observed complaints and past expected complaint levels influence future complaint counts. Consequently, the ACP model provides a useful framework for understanding and forecasting the temporal behavior of NPCC customer complaints.

Comparison of the performance of the INAR(1)-Poisson, INAR(1)-Negative Binomial, and ACP models

Table 4 presents the comparison of the performance for INAR(1)-Poisson, INAR(1)-Negative Binomial, and ACP models. The comparison was based on the loglikelihood, AIC and BIC model selection statistical tools used for measuring performance.

Table 4 Comparison of the performance of the INAR(1)-Poisson, INAR(1)-Negative Binomial, and ACP models

Model	Loglikelihood	AIC	BIC
INAR(1) Poisson	-345.21	694.42	702.15
INAR(1) Negative Binomial	-331.76	669.52	679.84
ACP(1,1)	-325.54	657.08	669.99

A comparison of the INAR(1)-Poisson, INAR(1)-Negative Binomial, and ACP(1,1) models was conducted using Log-Likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The ACP(1,1) model produced the highest Log-Likelihood (-325.54) and the lowest AIC (657.08) and BIC (669.99) values. Therefore, the ACP(1,1) model was selected as the most appropriate model for the Public Complaints Commission count series. The results indicate that incorporating both past observations and past conditional means improves model fit and better captures the serial dependence structure of the complaint counts.

One Year (January – December 2026) Forecast in ACP (1,1) with Poisson Innovation

Table 5 presents the monthly forecasts of NPCC customers' complaint counts for the period January to December 2026 generated using the Autoregressive Conditional Poisson [ACP(1,1)] model with Poisson innovations. The table also provides the corresponding 95% prediction intervals, which indicate the range within which future complaint counts are expected to fall with 95% confidence. The forecast results reveal a steady upward trend in the expected number of customer complaints throughout 2026. The predicted complaint count increases from 53,799 complaints in January to 64,640 complaints in December, representing an overall increase of approximately 10,841 complaints over the forecast horizon. This corresponds to a growth of about 20.2% during the year, suggesting that customer complaints are expected to continue rising if existing patterns persist.

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The month-to-month forecasts show a gradual and consistent increase in complaint volumes, indicating strong persistence in the complaint-generating process. This pattern is consistent with the ACP(1,1) model estimates, where the autoregressive parameter was found to be very close to one, implying that past complaint levels have a substantial influence on future complaint levels.

The 95% confidence intervals surrounding the forecasts are relatively narrow compared to the forecast values. For example, the forecast for January is 53,799, with a confidence interval of 52,308 to 55,322, while the forecast for December is 64,640, with a confidence interval of 64,120 to 65,147. The relatively tight intervals suggest a high degree of precision and confidence in the model's predictions.

The forecast indicates that NPCC is likely to experience continuously increasing customer complaint volumes throughout 2026. This projected growth may reflect persistent service challenges, increasing customer engagement, or expanding customer bases. From a managerial perspective, the results suggest the need for proactive measures to strengthen customer service operations, improve complaint resolution mechanisms, and allocate adequate resources to handle the anticipated increase in complaints. The ACP(1,1) model therefore provides valuable insights for planning and decision-making by highlighting the expected future trajectory of customer complaint counts.

Table 5 Forecast

Month	Forecast	95% CI (Lower)	95% CI (Upper)
January	53799	52308	55322
February	54695	53206	56192
March	55608	54156	57055
April	56539	55178	57933
May	57486	56090	58833
June	58452	57249	59718
July	59436	58319	60605
August	60438	59364	61521
September	61459	60481	62426
October	62500	61627	63341
November	63560	62846	64281
December	64640	64120	65147

DISCUSSION OF FINDINGS

The preliminary diagnostic tests (Box–Ljung and overdispersion tests) revealed significant autocorrelation and overdispersion in the NPCC complaint count series. This indicates that the data are not independently distributed over time and exhibit variability beyond the Poisson assumption. Consequently, three competing models were fitted: INAR(1)-Poisson, INAR(1)-Negative Binomial, and ACP(1,1). The results show strong persistence in complaint counts across all models, with autoregressive coefficients consistently close to one. The ACP(1,1) model provided the best fit based on log-likelihood, AIC, and BIC. Forecast results further indicated a steady upward trend in complaints for 2026. These findings align with the view that complaint processes are dynamic, persistent, and influenced by both past realizations and evolving expectations.

INAR(1) Model with Poisson Innovations

The INAR(1)-Poisson model revealed a highly significant autoregressive parameter ($\beta_1 = 0.982$), indicating strong temporal dependence in complaint counts. This supports the theoretical structure of INAR processes, where past observations directly influence current values through a thinning mechanism. Similar findings were reported by (Alzaid and Al-Osh, 1988), who introduced the INAR(1) framework and demonstrated its suitability for count data with autocorrelation. Likewise, Weiß (2008) emphasized that INAR models are appropriate for modeling persistent count processes such as accident or complaint data. However, the Box–Ljung test and overdispersion results suggest that the Poisson INAR(1) model may be inadequate due to its inability to handle excess variability. This agrees with (McKenzie, 2003), who noted that Poisson-based INAR models often underestimate dispersion in real-world applications.

INAR(1) Model with Negative Binomial Innovations

The INAR(1)-Negative Binomial model also showed strong persistence ($\beta_1 \approx 0.982$) but improved model fit over the Poisson version, as evidenced by lower AIC and BIC values. The inclusion of the overdispersion parameter ($\sigma^2 = 0.0779$) confirms that the complaint data exhibit extra-Poisson variation. This finding is consistent with (Böckenholt 1999; Jung and Tremayne 2006), who found that negative binomial INAR models are more appropriate for overdispersed count processes. Similarly, (Zhu and Joe, 2009) demonstrated that introducing negative binomial innovations significantly improves model flexibility in real-world count time series. Thus, the results support the argument that accounting for overdispersion leads to a more realistic representation of complaint dynamics.

Autoregressive Conditional Poisson (ACP) Model

The ACP(1,1) model produced the best fit among all models considered, with the highest log-likelihood and lowest AIC/BIC values. Both the autoregressive term ($\beta_1 = 0.978$) and the conditional mean effect ($\alpha_1 = 0.069$) were statistically significant, indicating that complaint counts depend not only on past observations but also on past expected values. This supports the findings of (Heinen, 2003), who introduced the ACP model and showed that it effectively captures time-varying conditional intensities in count data. Similarly, (Fokianos and Tjøstheim, 2011) demonstrated that conditional Poisson models outperform traditional INAR models when both serial dependence and conditional dynamics are present. The strong persistence observed in this study aligns with (Liboschik *et al.*, 2017), who noted that ACP-type models are particularly suitable for highly persistent count series.

Performance of INAR(1)-Poisson, INAR(1)-Negative Binomial, and ACP Models

Model comparison based on AIC, BIC, and log-likelihood indicates that the ACP(1,1) model outperforms both INAR specifications. The INAR(1)-Negative Binomial model performed better than the Poisson version, confirming the importance of addressing overdispersion.

These results are consistent with (Weiß, 2018), who reported that models incorporating both autoregressive structure and flexible innovation distributions tend to outperform simpler INAR models. Likewise, (Fokianos, 2012) emphasized that observation-driven models such as ACP often provide superior fit for count time series with complex dependence structures. Therefore, the findings support the general consensus in the literature that model performance improves when both serial dependence and overdispersion are jointly accounted for.

Forecast of Customers' Complaint Values (January–December 2026)

The ACP(1,1) forecasts indicate a steady increase in complaint counts throughout 2026, rising from 53,799 in January to 64,640 in December. This suggests a persistent upward trend in customer complaints, driven by strong autoregressive effects. This pattern is consistent with (Weiß, 2008), who noted that highly persistent count processes tend to produce smooth, trend-like forecast paths. Similarly, (Fokianos and Tjøstheim, 2011) observed that ACP-type models generate forecasts that reflect long memory and gradual adjustments rather than abrupt changes. From a practical standpoint, the upward forecast trend suggests worsening or unresolved service issues, consistent with findings in service quality studies such as (Parasuraman *et al.,* 1988), which link persistent complaints to gaps in service delivery and customer dissatisfaction. However, it should be noted that such strong persistence (β close to 1) may also indicate near-nonstationarity, a concern raised by (Wei, 2006) in time series analysis, implying that shocks to the system have long-lasting effects.

Across all models, the NPCC complaint series exhibits strong persistence and overdispersion, confirming that complaint dynamics are non-random and highly structured. While INAR models capture basic autocorrelation, the ACP model provides a more comprehensive representation by incorporating conditional dynamics. The literature strongly supports the superiority of ACP-type models for such data structures, and the forecasting results reinforce the expectation of increasing complaint volumes if current patterns persist.

CONCLUSION

This study examined and compared three count time series models for estimating and forecasting monthly customer complaints received by the Nigerian Public Complaints Commission (NPCC). The models considered were the first-order Integer-Valued Autoregressive model with Poisson innovations [INAR(1)-P], the first-order Integer-Valued Autoregressive model with Negative Binomial innovations [INAR(1)-NB], and the Autoregressive Conditional Poisson (ACP) model. The study was motivated by the need to appropriately model overdispersed and highly persistent complaint count data that violate the assumptions of classical time series methods. The first model, INAR(1) with Poisson innovations, was fitted to the complaint data and was found to capture the discrete nature of the series as well as its strong temporal dependence. The results showed a highly significant autoregressive coefficient close to unity, indicating strong persistence in complaint counts over time. However, the model was limited in handling overdispersion in the data, as it assumes equality between the mean and variance of the innovation process.

The second model, INAR(1) with Negative Binomial innovations, was introduced to address the limitation of overdispersion. This model allowed for an additional dispersion parameter, improving flexibility in capturing variability beyond the Poisson assumption. The results again revealed strong and significant serial dependence, while also confirming the presence of overdispersion in the complaint series. This made the model more suitable than the Poisson-based INAR specification, although challenges in fully capturing dynamic conditional structures still remained.

The third model, the Autoregressive Conditional Poisson (ACP), was developed to incorporate both past observed values and past conditional means into the modelling framework. This dual dependence structure allowed the model to better capture the evolving dynamics of complaint generation. The ACP model demonstrated strong statistical significance of all parameters and provided a more comprehensive representation of the temporal behaviour of the data compared to the INAR models. Finally, the study compared the forecasting performance of the three models for the period January to December 2026. The ACP model outperformed both INAR(1) specifications in terms of log-likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The forecast results from the ACP model indicated a steady upward trend in complaint volumes throughout 2026, suggesting increasing pressure on the complaint management system if current patterns persist.

Based on the findings of the study, it can be concluded that the monthly complaint data of the NPCC exhibit strong serial dependence, overdispersion, and persistent temporal dynamics. The INAR(1) model with Poisson innovations successfully captured the discrete and autoregressive nature of the data but was limited by its inability to fully accommodate overdispersion. The INAR(1) model with Negative Binomial innovations improved model flexibility by accounting for extra variability, making it more suitable for overdispersed count data.

The Autoregressive Conditional Poisson (ACP) model provided the most effective framework among the three models considered. Its ability to incorporate both past observations and past conditional expectations enabled it to better capture the complex dynamics underlying complaint generation. The model comparison results confirmed that the ACP model offered superior fit and forecasting accuracy. Furthermore, the forecast analysis for January to December 2026 revealed a consistent upward trend in complaint counts, indicating that complaints are expected to increase steadily if current conditions remain unchanged. This suggests that complaint generation is not random but driven by persistent systemic factors that continue to influence reporting patterns over time. The study concludes that ACP modelling is the most appropriate approach for analysing and forecasting NPCC complaint data due to its superior ability to handle overdispersion, temporal dependence, and dynamic behavioural patterns in count data.

Limitation

Despite the contributions of this study, certain limitations should be acknowledged:

i. The study is limited to monthly aggregated complaint data, which may mask finer temporal patterns observable in daily or weekly data.

ii. Only three models were considered, which may not capture the full range of modern count time series modelling approaches.

iii. The analysis assumes stationarity within the modelling period, which may not fully reflect structural changes or policy shifts over time.

iv. The study does not incorporate exogenous variables that may influence complaint generation, such as economic conditions or institutional reforms.

v. Forecast accuracy was evaluated based on statistical criteria only, without external validation using out-of-sample real-world observations beyond the dataset period.

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