Confidence Interval on Marginal Price and Capacity Adequacy of Power System with Price-Elastic Demand Using a Negative Binomial Random Variable of High Dispersion

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ABSTRACT

Based upon the Bernstein's inequality or the Gamma and Chi-square distributions as alternatives to the standard methods, we paraphrased a negative binomial distribution of high dispersion and transformed to a weibull distribution for probability evaluation. An increasing deployment of smart grid technologies and demand response programs has enabled demands to be more elastic in response to electricity prices, actively contributing to power system operation. However, demand-price elasticity could potentially impact accuracy of electricity price forecasting and generation capacity adequacy evaluation, which have not been thoroughly explored in literature. To this end, a confidence interval is developed to evaluate marginal electricity price and capacity adequacy of power systems with elastic demand, while considering uncertainties of both demand and supply sides (e.g., random outages of generating units with multi-bidding-segments as well as variability of renewable energy outputs). Specifically, based on the concept of marginal bidding segment, the probability of a bidding segment being marginal is computed via the probabilistic production simulation approach. Accordingly, the discrete probability distribution of marginal electricity price can be obtained, which is further used to calculate mean and variance of marginal electricity price with elastic demand. Moreover, considering the interaction between elastic demands and uncertain electricity prices, the loss-of-load probability and the expected unsaved energy are further estimated. Numerical studies are presented to validate effectiveness of the proposed method.

Keywords: Confidence interval, Capacity Adequacy, Demand-price Elasticity, Marginal price, Probabilistic Evaluation.

INTRODUCTION

The electric power industry has been experiencing restructuring and deregulation for the past two decades throughout the world, accompanied by an enlarged footprint, an increased generation capacity, and an intensified customer participation under the market environment. In this context, accurate electricity price forecasting plays an important role for market participants to construct effective bidding strategies and manage financial risks [Nowotarski & Weron, 2018]. In addition, different from regulated power systems in which sufficient generation capacities are usually secured by vertically integrated utilities, in deregulated electricity markets, economic and secure operations are driven by market competition between generating units. To this end, evaluating generation capacity adequacy in electricity markets is of great importance, especially when unplanned events such as random outage of generating units are considered.

Given a sample of n Independent, Identically distributed (i.id.) random variables with finite variance, the Central Limit Theorem (CLT) States that the distribution of the sample mean $\bar{X}$ is approximately Normal when the sample size n is large. As discussed in Rosenblum and Vander Laan (2008), the Normal approximation and a bootstrap method are standard techniques used in the construction of confidence interval for the mean μ even for moderately small sample size (e.g. n = 30). However, any application that n is large enough to render the difference between the distribution of $\bar{X}$ and the Normal in consequential. At moderate sample sizes, this CLT assumption cannot be assured in the case of random variables with highly skewed distributions (Wilcox, 2005). In particular, we will demonstrate that a sample mean constructed from *i.i.d.* Negative Binomial random variables of high dispersion exhibits a probability mass function with an extremely heavy right tail. Moreover, the variability of estimates of the standard error of $\bar{X}$ provides an additional degree of uncertainty. In practice, researchers whereby on a relatively small number of independent samples (such as the investigation of (Lloyd-Smith et al., 2005) should exercise caution to ensure that their conclusion are not greatly impacted by biased estimates of variability. Because $\bar{X}$ exhibits a skewed distribution, the Normal approximation may result in poor coverage and corresponding poor inferences. Similarly, the booststrap bias corrected and accelerated (BCA) method (Efron and Tibshirani, 1994) also relies upon imbedded normal theory that is impacted by skewness. This research will investigate the performance of those methods through simulation study and proposed a variety of improvements based upon Bernstein's inequality, a gamma model, and the chi-square (X^2) distribution for the construction of confidence intervals for the mean of Negative Binomial random variables of high dispersion.

Indeed, in emerging electricity market, interactions between demands and electricity prices as well as uncertainties of both demand and supply sides would further complicate the electricity price forecasting and the generation capacity adequacy evaluation, which is the main focus of this research.

Demand-price elasticity: In emerging power systems, customers are incentivized to adjust their electricity consumption behavior in response to electricity price signals [Weng et,al,2019], [Wang, et.al,2020]. This is described as a feature of demand-price elasticity [Pateraski, et.al, 2017], [Khan, et.al, 2016]. With a rapid upgrade of infrastructure and the deployment of various demand response programs, demand-price elasticity is being effectively enhanced.

However, like a coin has two sides, accompanied with new opportunities, elastic loads also pose challenges for both electricity price forecasting and generation capacity adequacy evaluation. As shown in Fig. 1, changes of demands and electricity prices interact with each other closely. Specifically, demands would respond to price signals, and such a response would further lead to price changes. This undoubtedly poses challenges to electricity price forecasting. In addition, compared with traditional capacity adequacy evaluation approaches, in which loads within a certain time period are treated as fixed values, elastic demand fluctuations in response to electricity prices could compromise generation capacity adequacy. As such, the interaction between demand and uncertain electricity price also poses challenges on generation capacity adequacy evaluation.

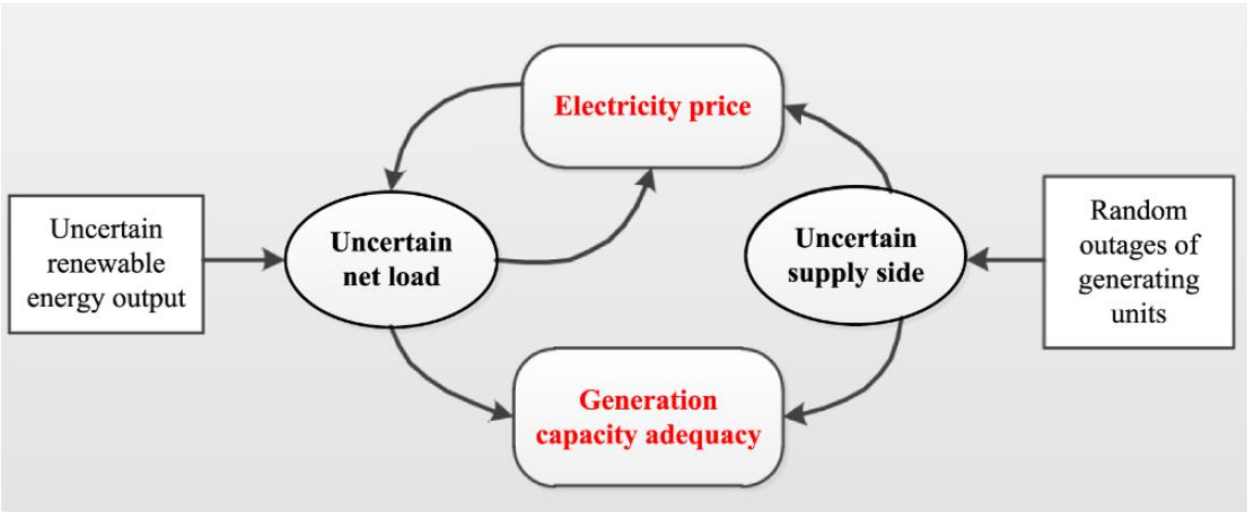

Fig. 1. Impacts of elastic demands and uncertainties on electricity price forecasting and generation capacity adequacy assessment in electricity market.

Uncertainties of demand and supply sides: Inevitable uncertainties of renewable energy resources could also significantly impact both electricity price forecasting and generation capacity adequacy evaluation [Carlo, et.al, 2016], [Toumas&Saddiqui,2017]. Specifically, with an integration of renewable energy resources, the total demand to be supplied by conventional generating units alters from the original load to net load. In this research, net load refers to as original load minus renewable outputs. Since electricity prices are derived through the market clearing process using bids from both supply and demand sides as random outages of generating units and uncertain generation output fluctuations of renewable resources could impact market clearing results, and further lead to large errors of electricity price forecast and inaccurate assessment on generation capacity adequacy.

The existing electricity price forecasting methods can be classified into three sets [Lago, et,al,2018], [Weron,2014]:

(1) ***Econometrics Methods:*** This set of methods are pioneered in the finance field, among which, autoregressive integrated moving average (ARIMA) model [Nowotarski&Weron,2016], [Vu,et.al,2019], [Rafiei,et.al,2017] and generalized autoregressive conditional heteroskedastic (GARCH) model [Garcia,et.al,2005] are widely used in electricity price forecasting. The parameters in these models can be estimated via historical data through statistical methods. However, it is difficult to explicitly consider the impacts of bidding rules and pricing mechanisms on electricity prices. To this end, some econometrics models are improved by taking market behaviors into consideration, such as the X-model proposed in references [Ziel&Steinert,2016], [Florian&Rick,2018]. However, although bidding behaviors of market participants could be reflected, they are still essentially based on the time series method which needs historical bidding information of buyers and sellers.

(2) ***Artificial Intelligence Based Methods:*** This set of methods is able to capture the input-output relationship even if the pricing mechanism is unknown. Artificial intelligence based methods include artificial neural network (ANN) [Panapakidis&Dagoumas,2016], [Singh,et.al,2017], support vector machine (SVW) [Shrivastava,et.al,2015], [Shayeghi,et.al,2016], weighted voting algorithm (WVM) [Wang,et.al,2019], and hybrid solutions of various algorithms [Yang,et.al,2017], [Ghayekhloo,et.al,2019]. These methods are considered as non-parametric models, and require a large number of historical price data.

(3) ***Structural Models:*** This set of methods forecasts electricity prices based on the bidding rules and pricing mechanisms of electricity markets. Structural models do not require historical electricity price data, and are easy to quantitatively analyze the influence of various factors on electricity price. For instance, game theory based structural models [Wang,et.al,2019], [Bompard,et.al,2010] consider strategic bidding behaviors of market participants. Moreover, some structural models consider uncertainties of supply and demand sides [Zhang&Li,2000], [Kang,et.al,2005], [Valenzuela&Mazumda,2007], [Siriruk&Valenzuela,2011]. Specifically, by leveraging marginal cost pricing method and inelastic load, probabilistic distribution of marginal price was formulated in [Zhang&Li,2000] to consider load uncertainty and random outages of generating units. In reference [Kang,et.al,2005], uncertainties of market participants' bidding behaviors are further considered, and the probabilistic distribution of marginal price is calculated based on the sequence operation theory. References [Valenzuela&Mazumda,2007], [Siriruk&Valenzuela,2011] forecast midterm and long-term price duration curves in the Cournot-competition-based power market while considering uncertainties of both supply and demand sides.

From above literature review we notice that, econometrics methods and artificial intelligence based methods heavily rely on historical price data over a long period. However, since the electricity market model is continuously improving, outdated historical price data may not reflect the up-to-date market practice and could compromise prediction accuracy. Moreover, although structural models do not require historical electricity price data, the existing models do not support probabilistic distribution calculation of marginal prices and are hard to explicitly consider demand-price elasticity.

With regard to generation capacity adequacy evaluation, generation capacity adequacy indices, such as loss-of-load probability (LOLP) and expected unserved energy (EUE), have been widely used to evaluate power system reliability [Wang,et.al,2020]. These indices, however, do not properly consider demand-price elasticity under a market environment. Very few studies considered the impact of demand-price elasticity on generation capacity adequacy. For instance, [Yu,et.al,2005] modeled the interruptible load as an equivalent price-based generation resource in the capacity adequacy evaluation. Reference [Yang,et.al,2019] presented a reliability evaluation method considering time-of-use electricity pricing. Indeed, in this literature, the capacity adequacy evaluation assumes demands with respect to deterministic electricity prices. However, in the market environment with inevitable uncertainties and the interaction between demand and electricity price, demand fluctuations would constantly respond to uncertain electricity prices. As such, the capacity adequacy evaluation approaches in literature could be invalidated.

In summary, demand-price elasticity, although would significantly impact electricity price forecast and capacity adequacy evaluation, has been widely neglected in many existing studies. In this research, a reliable confidence interval based on probabilistic approach is established to estimate marginal prices while considering demand-price elasticity. This approach is further integrated into the proposed capacity adequacy evaluation method while properly considering both elastic demands and uncertain electricity prices.

The Negative Binomial Distribution

A negative Binomial distribution is conventionally used to compute the probability that the total of k failures will result before the r^{th} success is observed when each trial is independent of all others and results in success with a fixed probability p. As described in Hilbe (2007), a Negative Binomial distribution may instead be parameterized in terms of a mean parameter $\mu = r\left(\frac{1}{p} - 1\right)$ and a dispersion parameter $\theta = r$. (We will adopt this alterative parametrization for the remainder of this paper). Then, for any $\mu \in \mathbb{R}^+$ and $\theta \in \mathbb{R}^+$, the resulting probability mass function for the Negative Binomial random variable $X \sim NB(\mu, \theta)$ is

$$P(X = k) = \frac{\mu^k}{k!} \frac{\Gamma(\theta + k)}{\Gamma(\theta)[\mu + \theta]^k} \frac{1}{\left(1 + \frac{\mu}{\theta}\right)^n}, \mathrm{k} \in \mathbb{Z}^+. \quad (1)$$

Equation (1) can be shown to converge to the probability mass function of a Poisson random variable with mean parameter μ as $\theta \to \infty$ (Hilbe, 2007). For this reason, the Negative Binomial may be considered by the value of θ. Negative Binomial models are useful as robust alternatives to the Poisson that allow the variance parameter to exceed the mean. For instance, smaller values of θ result in a higher dispersion by adding more weight to the right tail of the probability mass functions, which necessarily results in a higher variance. When the value of $\theta$is very small, the Negative Binomial distribution exhibits a high degree of skewness. As a result of the extreme dispersion of the Negative Binomial from the Poisson in this case, the sample mean $\bar{X}$ of n i.i.d. $N\,B(\mu, \theta)$ observations may not be reasonably close to the Normal in distribution for small values of n. Wilcox (2005) warns that standard confidence intervals based upon a Normal approximation may result in poor coverage in scenarios such as this.

Probabilistic models of net load and supply

Probabilistic models of the net load

In an electricity market with stepwise bidding curves of generators, electricity prices would change in discrete jumps. Similar to the bidding curves of generators, one may also simulate price-demand elasticity via a step-wise curve, which corresponds to a piecewise linear utility function of the demand side. In this paper, we consider a quadratic utility function of the demand side. To this end, Eq. (1.1) can be derived from the inverse demand function which is obtained based on the first order derivation of the utility function. Load *Dt* with demand-price elasticity is represented by a linear demand function as in (1.1) [27,28]. In Eq. (1.1), *Lt* is initial load at time period *t*; *bt* is electricity price at time period *t; e* denotes nonnegative elasticity coefficient, indicating that, like many other commodities, electric load decreases with an increase in electricity market price. Such a linear model also facilitates our calculations when random outages of generating units with multi-bidding-segments and variability of renewable energy outputs are further considered.

$$D_t = L_q - e.b_t \quad (1.1)$$

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Uncertainties of renewable energy resources need to be considered as it will propagate into the net load profile. Taking wind power uncertainty as an example, uncertain wind speed could be described via a Weibull distribution [32]:

$$f(v,\lambda,k) = \begin{cases} \frac{k}{\lambda}\left(\frac{v}{\lambda}\right)^{k-1} \quad e^{-(v/\lambda)^k}, & v \geq 0 \\ 0, v < 0 \end{cases} \tag{2}$$

where k and λ are respectively shape parameter and scale parameter, and v is wind speed. k and λ depend on the feature of wind speed time matrix and average wind speed. NS scenarios of wind speeds are generated from the Weibull distribution to simulate wind speed uncertainty, each of which has the probability of $1/NS$. A recursive backward scenario reduction technique [33] is employed to reduce the number of wind speed scenarios from NS to the desired number N. This method is an iterative process. In each iteration, the Kantorovich distance for each pair of scenarios in the current scenario set is computed; backward search is conducted to identify the wind speed scenario with the minimum Kantorovich distance; the product of the minimum Kantorovich distance and probability of the assessed scenario is recorded in the set $\boldsymbol{QKD}$, and the scenarios (one or multiple) in $\boldsymbol{QKD}$ with the minimum value are eliminated. This procedure is executed iteratively until N scenarios are left, which are used to simulate random wind speeds for evaluation.

With simulated wind speeds in each scenario, the corresponding wind power output can be calculated via Eq. (3).

$$0{,}0 \leq v_{n,i,1} \leq v_i CI$$

$$P^W_{n,i,1} = \begin{cases} 0.5\rho A v^3_{n,i,1} C^p_i, v^a_i \leq v_{n,i,1} \leq v^R_i \\ P^R_i, v^R_i \leq v_{n,i,1} \leq v^{CO}_i \end{cases} \tag{3}$$

$$0, v_{n,i,1} \geq v^{CO}_i$$

where $P^W_{n,i,1}$ is power output of the ith wind generator at time period t in scenario n, P^R_i is rated power output, ρ is air density, A is windward area, C^P_i is rotor power coefficient, v^R_i, v^{CI}_i, and v^{CO}_i are respectively rated, cut in, and cut-out wind speeds. Accordingly, N scenarios of wind power outputs can be generated, denoted as $P^W_{n,i,1}$. The probability of scenario n is $P^W_{n,i,1}$. Assuming a wind farm is composed of nw wind turbine generators with the same parameters (i.e. identical cut-in, rated, and cut-out wind speeds), the total power output under scenario n can be simply calculated as:

$$P^W_{n,i} = \sum_{i=1}^{nw} P^W_{n,i,1}, n = 1,2,\ldots,N \tag{4}$$

As such, simultaneously considering uncertain wind power output and demand-price elasticity, the net load of scenario n at time period t, which is the load minus the wind generation of scenario n at time period t, can be calculated as:

$$D_{n,i} = L_i - e.b_i - P^W_{ni}, n = 1,2,\ldots,N$$

Probabilistic models of the supply side

Generating units could potentially suffer from random outages during operation, which should be taken into account when modeling the supply side. Forced outage rate has been widely used in the long-term reliability evaluation process to represent the unavailability of generating units in the steady state, which may not be proper for short-term studies such as electricity price forecasting. As such, the outage replacement rate [34-35] is utilized, which describes the probability that a generating unit fails and cannot be replaced during the lead time *t*. The outage replacement rate *qm*,*t* of generating unit *m* with the lead time *t* is described as follows:

$$q_{m,i} = \frac{\lambda_m}{\lambda_m + \mu_m} + \frac{\exp[-(\lambda_m + \mu_m)t]}{\lambda_m + \mu_m}[\mu_m U(0) - \lambda_m A(0)] \qquad (6)$$

where λ_m and μ_m are the failure rate and the repair rate of generating unit *m*. The outage replacement rate is a time-varying quantity, depending on the leading time *t* and the current operation status. If the unit is available at the initial time point *t* = 0, *A*(0)=1 and *U*(0)=0; if the unit is unavailable at *t* = 0, *A*(0) = 0 and *U*(0) = 1. It can be seen from Eq. (6) that the steady state (leading time *t*→∞) of $q_{m,t}$ is $\frac{\lambda_m}{\lambda_m+\mu_m}$, which is equal to the forced outage rate of generating unit *m*.

In this paper, monotonically increasing step-wise bidding curves [36] of generating units, which are commonly used in practical electricity markets, are considered. Specifically, without loss of generality, the bid curve submitted by each generating unit *m* contains *K* pairs of bidding quantities and prices. The first bidding segment of unit *m* starts at the minimum capacity P_m^{min}, and contains the quantity of P_m^{min} offered at price b_m, 1. The *k*th (for *k* = 2, …, *K*) bidding segment consists of a quantity $P_{m,k}$ offered at price $b_{m,k}$. The total capacity of *K* bidding segments submitted by generating unit *m* is P_m^{max}, the maximum capacity of unit *m*. It is emphasized that bidding prices of the *K* bidding segments are in an ascending order, i.e., $b_{m,1} \leq b_{m,2} \leq \cdots \leq b_{m,K}$.

Considering the marginal pricing scheme widely used by ISOs, electricity prices will be cleared at the bidding price of the marginal bidding segment. To this end, we can sort bidding segments of all generating units in ascending order of their bidding prices. This economic merit order is used to determine optimal dispatch levels of generating units. An illustrative example of two generating units with *K* = 2 is shown in Fig. 2. Bidding segments of the two generating units are sorted in ascending order. Marginal price at the load level of *Dt* in Fig. 2 is $b_{1,2}$.

In the traditional market clearing models [25], operation status of a generating unit is assumed to follow a two-state model: available state and unavailable state (i.e., random outage). Under the available state, unit *m* can provide the maximum capacity P_m^{max}. Considering the multi-bidding segments submitted by a generating unit, under the available state, unit *m* can be operated in *K* capacity states including: the minimum capacity P_m^{min}, the maximum capacity P_m^{max}, and (*K* − 2) states

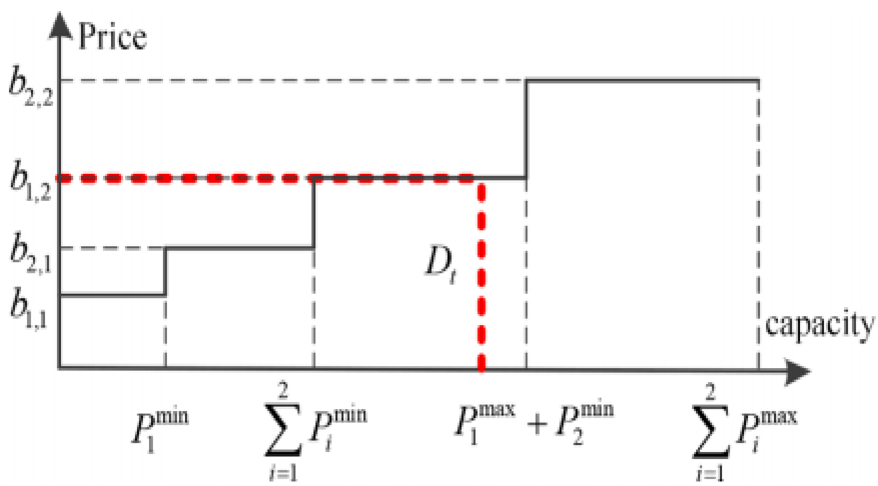

Fig. 2. Aggregated step-wise bidding curves of two generating units.

$P_m^{min} + \sum_{i=2}^{k} P_{m,i}$, $(k = 2, \ldots, K-1)$ in between. As such, the operation state of each unit follows a (*K* + 1)-state model to describe the available capacity: *K* available states and an unavailable state. This can be

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expressed as in (7)-(9), where $x_{m,1}$ and $x_{m,K}$ are the minimum and the maximum available capacities of unit $m, x_{m,k}$ is available capacity of unit *m* after the *k*th bidding segment is loaded.

$$x_{m,I} = \begin{cases} P_m^{min}, & with\ probability: 1 - q_{m,t} \\ 0, & with\ probability:\ q_{m,t} \end{cases} \tag{7}$$

$$x_{m,k} = \begin{cases} P_m^{min} + \sum_{i=2}^{k} P_{m,i}, & with\ probability : 1 - q_{m,t} \\ 0, & with\ probability : q_{m,t} \end{cases}, k = 2, \dots, K-1 \tag{8}$$

$$x_{m,K} = \begin{cases} P_m^{max}, & with\ probability: 1 - q_{m,t} \\ 0, & with\ probability:\ q_{m,t} \end{cases} \tag{9}$$

For a power system with *I* generating units, we denote the bidding segments {*j* = 1,2,…,*I*, *I* + 1,…,*KI*} at the economic merit loading order. We further use A_j to denote the total available capacity after the first *j* segments are loaded, i.e., where *mj* represents that the *j*th segment in the economic merit loading order belongs to unit *m*.

$$A_j = A_{j-1} + x_j, j = 1, \dots, KI \tag{10}$$

$$x_j = \begin{cases} x_{m_j.k}, & k = 1 \\ x_{m_j.k} - \sum_{i=1}^{k-1} x_{m_j,i}, & k = 2, \dots, K \end{cases} \tag{11}$$

The probabilistic production simulation technique is utilized to characterize availabilities of generating units. Considering that available capacities of individual units are independent, using the Z transform method of probabilistic production simulation technique, the probability distribution function of the system's total available capacity can be obtained. Based on the general formulation of discrete probability distribution function, the discrete cumulative probability distribution function of the system's total available capacity *x* after loading the first *j* segments can be formulated as follows:

$$F_{A_j}(x) = \sum_{i=1}^{Q_j} P_{j,i}.u\left[x - A_{j.i}\right], j = 1,2, \dots, KI \tag{12}$$

where $A_{j,i}$ and $P_{j,i}$ denote the *i*th available capacity state and its probability after the first *j* segments are loaded, respectively. *Qj* is the number of available capacity states after the first *j* segments are loaded. The available capacity states are arranged in a strictly ascending order, i.e. $A_{j,i} < A_{j,i+1}$. $u(x)$ is the unit step function.

Note that the classic recursive convolution method [23-25] used in literature is an approximation method. In this paper, we use Z transform method to rigorously model the inherent discrete characteristics of probability distributions of available generation capacity. As such, it is an exact method in principle.

Considering random outages of units and uncertainties of net loads, generation capacity shortage could happen. To effectively consider this, a hypothetical bidding segment (i.e., the (*KI* + 1)th bidding segment) is added to calculate unserved energy when all generation segments are loaded, i.e., $P_{KI+1}^{max} = \infty$ and $q_{KI+1,t} = 0$. The bidding price of the (*KI* + 1)th segment b_{KI+1} is the value of the lost load. Generally, $b_{KI+1} > b_i, i = 1,2, \dots, KI$.

Evaluation of marginal price and capacity adequacy with elastic demand

Probability distribution of marginal bidding segment

The marginal bidding segment in the cumulative loading order to meet the load at time period *t* is denoted as *Jt*. When marginal price is used as the market clearing price, the market price at time *t* equals to the bidding price of the marginal bidding segment, i.e. $b_t = b_{J_t}$. For a given system, J_t depends on uncertain net load level and generation availability, thus is a random variable. To this end, to model the stochastic behavior of electricity price, we need to determine distribution of the marginal bidding segment, i.e. the distribution of J_t.

First, when neglecting renewable energy resources and demand price elasticity, D_t equals to L_t. If bidding segment j is the marginal bidding segment, then $A_j > D_t$. Furthermore, if $A_j > D_t$, then $\forall i > j, A_i > D_t$. As such, the marginal bidding segment is the minimum *j* that satisfies $A_j > D_t$. This can be expressed as:

$$J_t = \min\{j | A_j > D_t\} \tag{13}$$

When demand-price elasticity is considered, D_t varies with the marginal price. Marginal price b_t at time *t* is one of the bidding prices of the (*KI* +1) bidding segments. Specifically, if the marginal bidding segment is segment *j*, then $b_t = b_j$, and the load at time period *t* is:

$$D_{j,t} = L_t - e.b_j, j = 1,2,\dots,KI + 1 \tag{14}$$

Moreover, if the marginal bidding segment is segment *j*, then $A_j > D_{j,t}$. Meanwhile, the biding segments are dispatched in a merit order, i.e., $b_i \le b_{i+1}, i = 1, 2, \dots, KI$. As such, the marginal bidding segment can be expressed as:

$$J_t = min\{ j | A_j > D_{j,t}\} \tag{15}$$

To this end, the probability distribution of the marginal bidding segment with demand-price elasticity can be described by the following two events: *Event 1* representing "the total available capacity of the first (*j*-1) segments is less than or equal to $D_{j-1,t}$", and *Event 2* representing "the total available capacity of the first *j* segments is larger than $D_{j,t}$". That is, if *Event 1* and *Event 2* occur simultaneously, segment *j* is the marginal bidding segment in time *t*. The probability $g_{j,t}$ of both events occurring simultaneously can be expressed as:

$$g_{j,t} = \Pr\{A_{j-1} \le D_{j-1,t}, A_j > D_{j,t}\} = \Pr\{A_{j-1} \le D_{j-1,t}\} + \Pr\{A_j > D_{j,t}\} - \Pr\{A_{j-1} \le \cup A_j > D_{j,t}\}, j = 1,2,\dots,KI + 1 \tag{16}$$

Note that as at least one of *Event 1* and *Event 2* will occur, according to the probability theory, $Pr\{A_{j-1} \le D_{j-1,t} \cup A_j > D_{j,t}\} = 1$. Accordingly, (16) can be rewritten as in (17), which can be equivalently converted into (18)

$$g_{j,t} = Pr\,\{A_{j-1} \le D_{j-1,t}\} + Pr\,\{A_j > D_{j,t}\} - 1 \tag{17}$$

$$g_{j,t} = Pr\,\{A_{j-1} \le D_{j-1,t}\} - Pr\,\{A_j > D_{j,t}\} = F_{Aj-1}[D_{j=1,t}] - F_{Aj}[D_{j,t}] \tag{18}$$

By further considering the initial and terminal conditions of $F_{Aj}\,[D_{j,t}], i.e., F_{A0}\,[D_{0,t}] = 1$ and $F_{AKI+1}\,[D_{KI+1,t}] = 0$, probability distribution of the marginal bidding segment at time period *t* can be finally expressed as:

$$\begin{cases} g_{1,t} = \Pr\{A_1 > D_{1,t}\} = 1 - F_{A1}\,[D_{1,t}] \\ g_{j,t} = \{F_{A_{j=1}}[D_{j-1,t}] - F_{Aj}[D_{j,t}]\}\ , j = 2,\dots,KI \\ g_{KI+1,t} = F_{A_{KI}}\,[D_{KI,t}] \end{cases} \tag{19}$$

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If we further consider demand-price elasticity and uncertain wind output simultaneously, the probability distribution of marginal bidding segment at time *t* can be obtained as:

$$\begin{cases} g_{1,t} = 1 - \sum_{n=1}^{N} P_{n,t}^{wind} \, . F_{A1}\left[D_{1,n,t}\right] \\ g_{j,t} = \left\{F_{A_{j=1}}\left[D_{j-1,n,t}\right] - F_{Aj}\left[D_{j,t}\right]\right\} \sum_{n=1}^{N} P_{n,t}^{wind} \, , j = 2, \dots, KI \\ g_{KI+1,t} = \sum_{n=1}^{N} P_{n,t}^{wind} \, . F_{A_{KI}}\left[D_{KI,n,t}\right] \end{cases} \quad (20)$$

Estimation of marginal price

Since marginal price is equal to the bidding price of the marginal bidding segment, the probability distribution of *bt* can be derived via (20). The distribution of discrete marginal price at time period *t* can be written as:

$$Pr(X = b_i) = g_{i,t}, i = 1, 2, \dots, KI + 1 \quad (21)$$

For each $g_{i,t}, F_{Aj}\left[D_{j,n,t}\right]$ in (20) can be calculated as follows:

$$F_{Aj}\left[D_{j,n,t}\right] = \sum_{m=1}^{M_{j,n,t}^{*}} P_{j,m} \quad (22)$$

Where $M_{j,n,t}^{*}$ can be obtained as follows:

$$A_j\left(M_{j,n,t}^{*}\right) \le D_{j+1,n,t} < A_j\left(M_{j,n,t}^{*} + 1\right) \quad (23)$$

Accordingly, the probability density function of the marginal price at time period *t* can be written as:

$$PDF(x) = \sum_{i=1}^{KI+1} g_{i,t} \delta(x - b_t) \quad (24)$$

Where $\delta(x)$ is the unit-impulse function. It can be verified via (20) that

$$\sum_{i=1}^{KI+1} g_{i,t} = 1 \quad (25)$$

which means (20) is a mathematically reasonable density function. The mean and variance of the marginal price, $E[b_t]$ and $Var[b_t]$, can be calculated as follows:

$$E[b_t] = \sum_{i=1}^{KI+1} g_{i,t} b_t \quad (26)$$

$$Var[b_t] = \sum_{i=1}^{KI+1} g_{i,t} b_i^2 - \{E[b_t]\}^2 \quad (27)$$

Estimation of capacity adequacy

Considering demand-price elasticity, the interplay between demand and electricity price would lead to load changes. Thus, load within a certain time period is not a fixed value. To this end, when hourly electricity price follows a discrete distribution, loads in response to each possible electricity price can be evaluated.

As electricity price is one of the bidding prices of the (KI +1) bidding segments, two situations can be divided as follows: *Situation 1*: when electricity price $b_t \in \{b1, b2, \ldots, bK_I\}$, the bidding segment $Jt \in \{1, 2, \ldots, KI\}$ and the net load with demand-price elasticity is $D_{j,t}$. To this end, the total available capacity of the first j segments is larger than $D_{j,t}$.

Meanwhile, load loss would happen only when the net load is greater than the available capacity. As such, when the marginal bidding segment Jt is one of the first KI segments, load loss will not happen.

Situation 2: when electricity price $bt = bKI$+1, the bidding segment $J_t = KI + 1$ and the net load with demand-price elasticity is $D_{KI+1,t}$. Thus, the total available capacity of the first KI segments is insufficient to cover $D_{KI+1,t}$ and load loss would happen.

As such, considering price-elastic demand as well as uncertain wind output, the loss-of-load probability can be calculated via the probability that the hypothetical bidding segment (i.e., the $(KI + 1)^{\text{th}}$ segment) is the marginal bidding segment. It can be derived as follows:

$$LOLP_t = g_{KI+1,t} - \sum_{n=1}^{N} P_{n,t}^{wind} \,.\Pr\{A_{KI} = D_{KI,n,t}\} = \sum_{n=1}^{N} P_{n,t}^{wind} \,.\Pr\{A_{KI} < D_{KI,n,t}\} \qquad (28)$$

Correspondingly, the expected unserved energy of the system is the expected quantify of the $(KI + 1)^{\text{th}}$ segment. Indeed, the expected quantity of the jth segment $EE_{j,t}$ equals to the expected unserved energy after only the first $(j - 1)$ segments are loaded minus the expected unserved energy after the first j segments are loaded, i.e.,

$$EE_{j,t} = EUE_{j-1,t} - EUE_{j,t} \qquad (29)$$

where $EUE_{j,t}$ is the expected unserved energy after only the first j segments are loaded at time period t, which can be calculated as follows:

$$EUE_{j,t} = \Delta t \sum_{n=1}^{N} P_{n,t}^{wind} \,.\sum_{m=1}^{M_{j,n,t}} \left[D_{j+1,n,t} - A_{j,m}\right] P_{j,m} \qquad (30)$$

Where, $M_{j,n,t}$ can be obtained as follows:

$$A_j(M_{j,n,t}) < D_{j+1,n,t} \le A_j(M_{j,n,t} + 1) \qquad (31)$$

As the(KI + 1)th segment is sufficiently large, the expected unserved energy after loading all KI + 1 segments is zero. As such, the expected unserved energy of the system equals to the expected unserved energy after only loading the first KI segments, i.e.,

$$EUE_t = EE_{KI+1,t} = EUE_{KI,t} \qquad (32)$$

Furthermore, considering elastic demand as well as uncertain wind output, LOLP and EUE at time period t can be expressed as:

$$EUE_t = \Delta t \sum_{n=1}^{N} P_{n,t}^{wind} \,.\sum_{m=1}^{M_{KI,n,t}} \left[D_{KI+1,n,t} - A_{KI,m}\right] P_{KI,m} \qquad (33)$$

$$LOLP_t = \sum_{n=1}^{N} P_{n,t}^{wind} \,.\sum_{m=1}^{M_{KI,n,t}} P_{KI,m} \qquad (34)$$

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Where $M_{KI,n,t}$ can be obtained as follows:

$$A_{KI}(M_{KI,n,t}) < D_{KI+1,n,t} \leq A_{KI}(M_{KI,n,t}+1) \tag{35}$$

Numerical results

Description of the test system

In order to verify the effects of demand-price elasticity and wind power integration on marginal price and generation capacity adequacy, numerical simulations based on the annual peak load day of a practical system in Nigeria are conducted and data from Nigeria Energy Commission were used. In this test system, the ratio of the maximum original load over the installed conventional capacity is 0.9897. Table I shows the 10 conventional generating units in their loading order, assuming that all units are available at time t = 0. The lead time is 24h. Based on the failure rate and the repair rate shown in Table I, the outage replacement rate of each generating unit can be calculated via Eq. (6). Accordingly, the random outages of generating units are introduced. The number of bidding segments is set as K=2. b_{KI+1} is set as 100$/MWh. With an exchange rate of N1347 to 1$.

The wind speed regime is characterized by the Weibull distribution, with the Weibull parameters k and λ of 10.7998 and 5.7974, while the average and standard deviation (SD) of wind speed are 10 and 2 m/s. 3000 random scenarios of wind speeds are generated based on the Weibull distribution, each of which is assigned with an equal probability of 1/3000. Then, a recursive backward scenario reduction technique [33] is employed to reduce the number of wind speed scenarios to 50 representative scenarios. The relationship between wind turbine's output and wind speed is described as PW = 0.7646v3, with the cut-in, rated, and cut-out wind speeds of 2, 11, and 20 m/s, respectively. The standard deviation (SD) of wind speed is 2 m/s. Wind speed and the initial load data of 24 h are shown in Fig. 3. Fig. 4 shows the probabilities and wind power outputs of 50 scenarios, with mean values of wind speed of 10 m/s and the ratio of installed wind over conventional capacity of 0.11.

Table 1 Parameters for generating units.

Unit	$P_m^{max}(MW)$	$\lambda_m^{-1}(h)$	$\mu_m^{-1}(h)$	P_m^{min}/P_m^{max}	$b_{m,1}$ (\$/MWh)	$b_{m,2}$ (\$/MWh)
1	1000	1440	160	50%	0.45	4.50
2	900	1300	150	50%	0.50	5.00
3	700	1200	130	50%	0.55	5.50
4	600	1100	110	50%	0.575	5.75
5	500	1150	110	45%	0.60	6.00
6	400	1100	90	45%	0.85	8.50
7	300	1000	70	45%	1.00	10.00
8	200	850	48	40%	1.45	14.50
9	100	600	48	40%	2.25	22.50
10	50	350	12	40%	4.40	44.00

Impacts of demand-price elasticity on marginal price and capacity adequacy

To verify the effect of demand-price elasticity on marginal price and generation capacity adequacy, numerical simulations with the price elasticity coefficient of 0, 15, 30 MW($/MWh)$^{-1}$ are conducted. With respect to the

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same electricity price change, a larger elasticity coefficient leads to higher load changes. Fig. 5 shows the mean and variance of marginal price under different elasticity coefficients. It can be noted that the mean and variance of marginal price decrease with a higher elasticity coefficient in all 24 h. The reduction of marginal price is 23.24 \$/MWh at 19:00 (the peak-load hour) with the elasticity coefficient increasing from 0 to 30 MW(\$/MWh)$^{-1}$. The reduction of marginal price is only 0.11\$/MWh at 4:00 (the minimum loading hour) with the elasticity coefficient increasing from 0 to 30 MW(\$/MWh)$^{-1}$. This clearly shows that the reduction in marginal price caused by the demand-price elasticity is more significant in higher load periods. The change in variances of marginal prices follows the same trend. Consequently, with the increase in demand-price elasticity, the marginal price and its variance can be reduced, i.e., demand-price elasticity can smooth out the marginal price volatility. In addition, the influence is more significant during heavier load periods.

The impact of demand-price elasticity on generation capacity adequacy indices is shown in Fig. 6. It can be seen from Fig. 6 that with the increase in elasticity coefficient, LOLP and EUE in 24 h are both decreased. The reduction of LOLP is 0.1641 at 19:00 with the elasticity coefficient increasing from 0 to 30 MW(\$/MWh)$^{-1}$. The reduction of

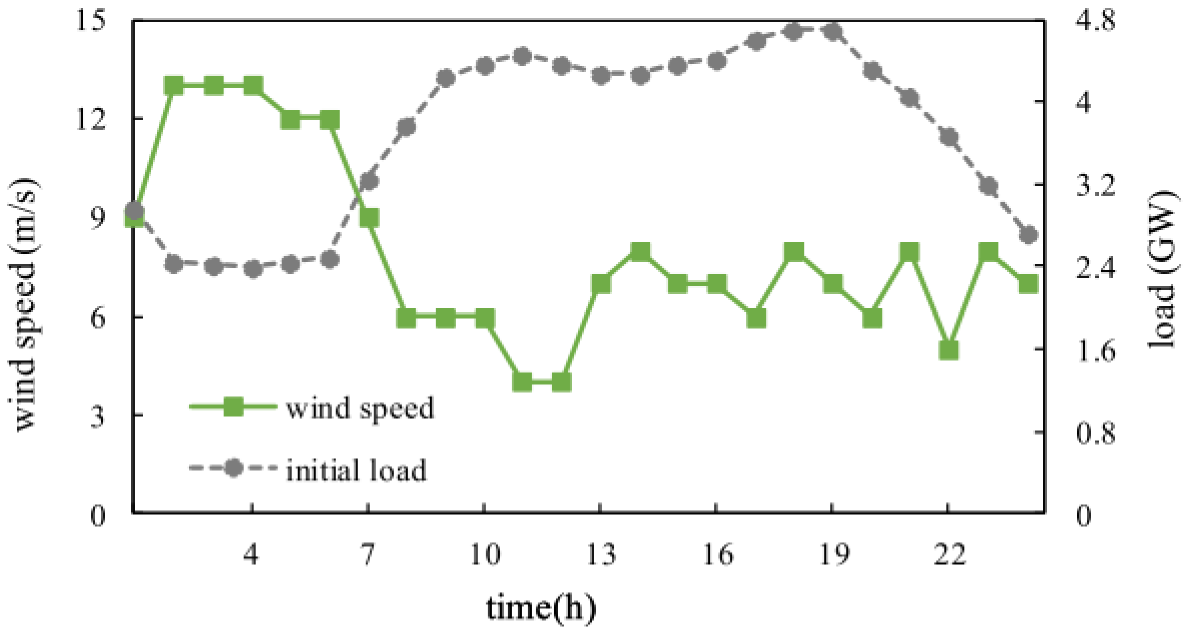
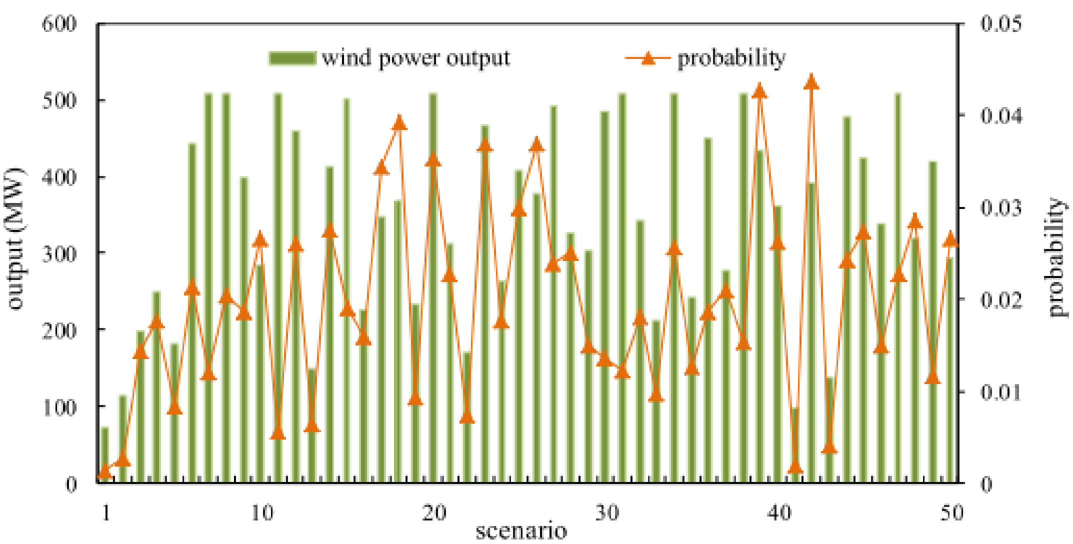


Fig. 3. Data of initial load and wind speed of 24 h.



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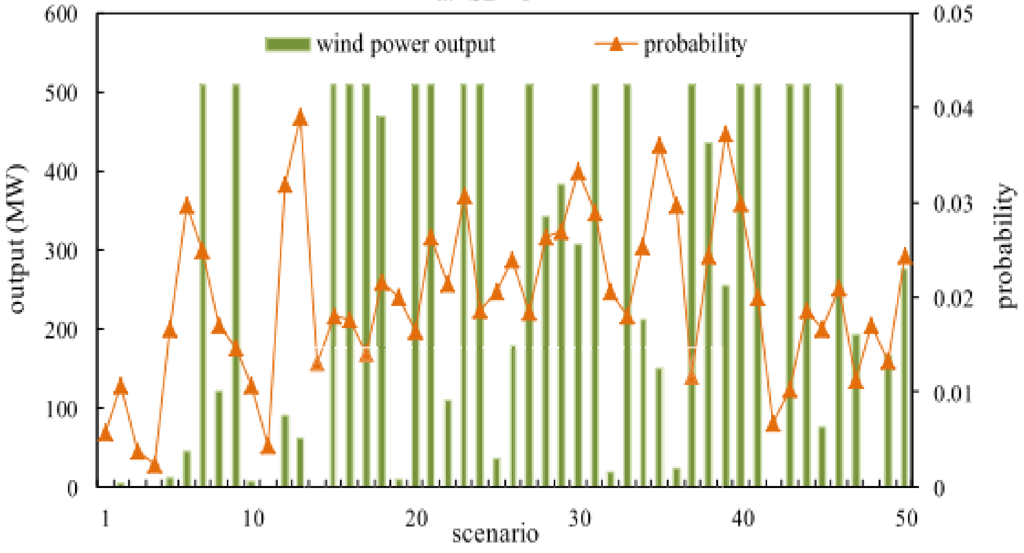
a. SD=1



b. SD=5

Fig. 4. Scenarios probabilities and outputs of wind power under different SDs of wind speed.

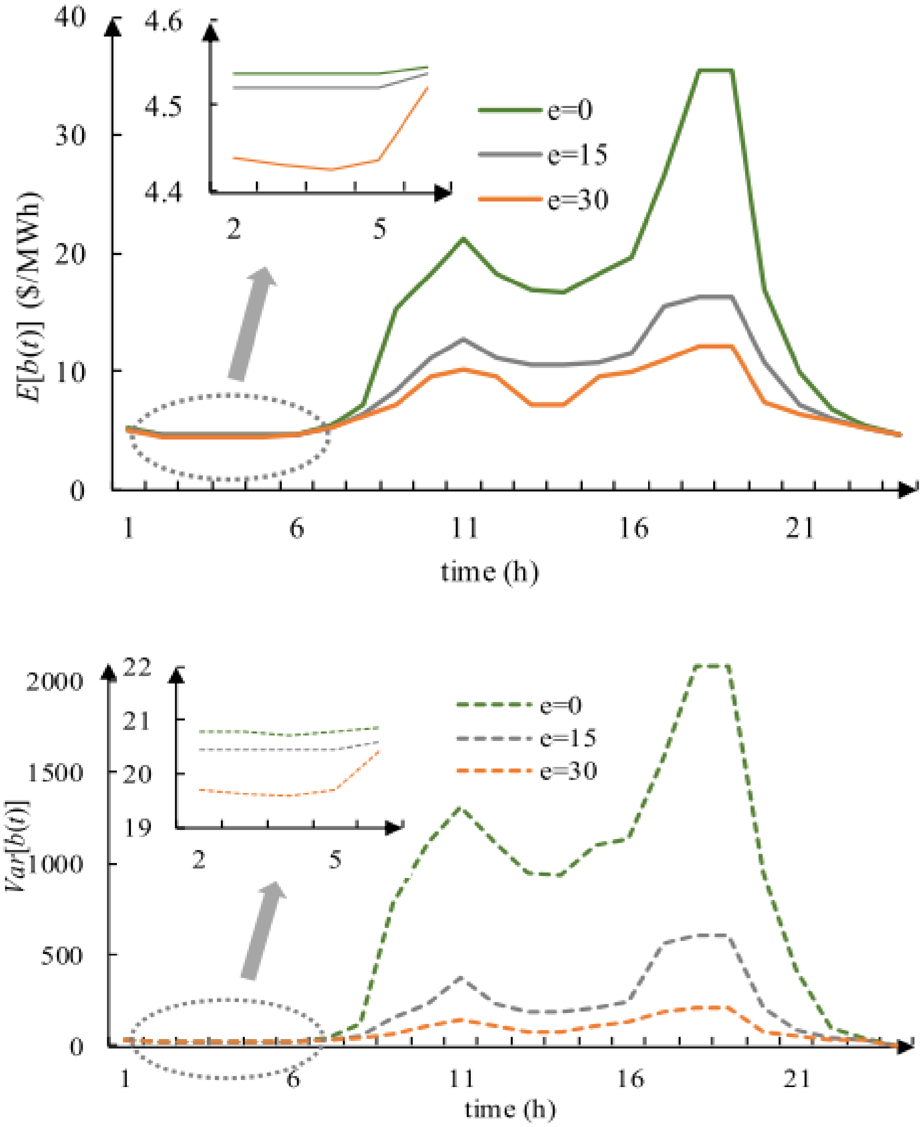


Fig. 5. Mean and variance of marginal price against different elasticity coefficients.

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LOLP is only 1.16×10^{-5} at 4:00 with the elasticity coefficient increasing from 0 to 30 MW($/MWh)$^{-1}$. This clearly shows that the reduction in LOLP caused by the demand-price elasticity is more significant in higher load periods. The change in EUE follows the same trend. That is, the generation capacity adequacy can be effectively improved with the increase in demand-price elasticity. Similarly, it can be seen that the influence is more significant during heavier load periods.

Effect of load on marginal price and generation capacity adequacy

To assess the influence of load on hourly marginal price and generation capacity adequacy, a sensitivity analysis with respect to different load levels ranging from 2.5 GW to 4.75 GW is shown in Fig. 7. As the

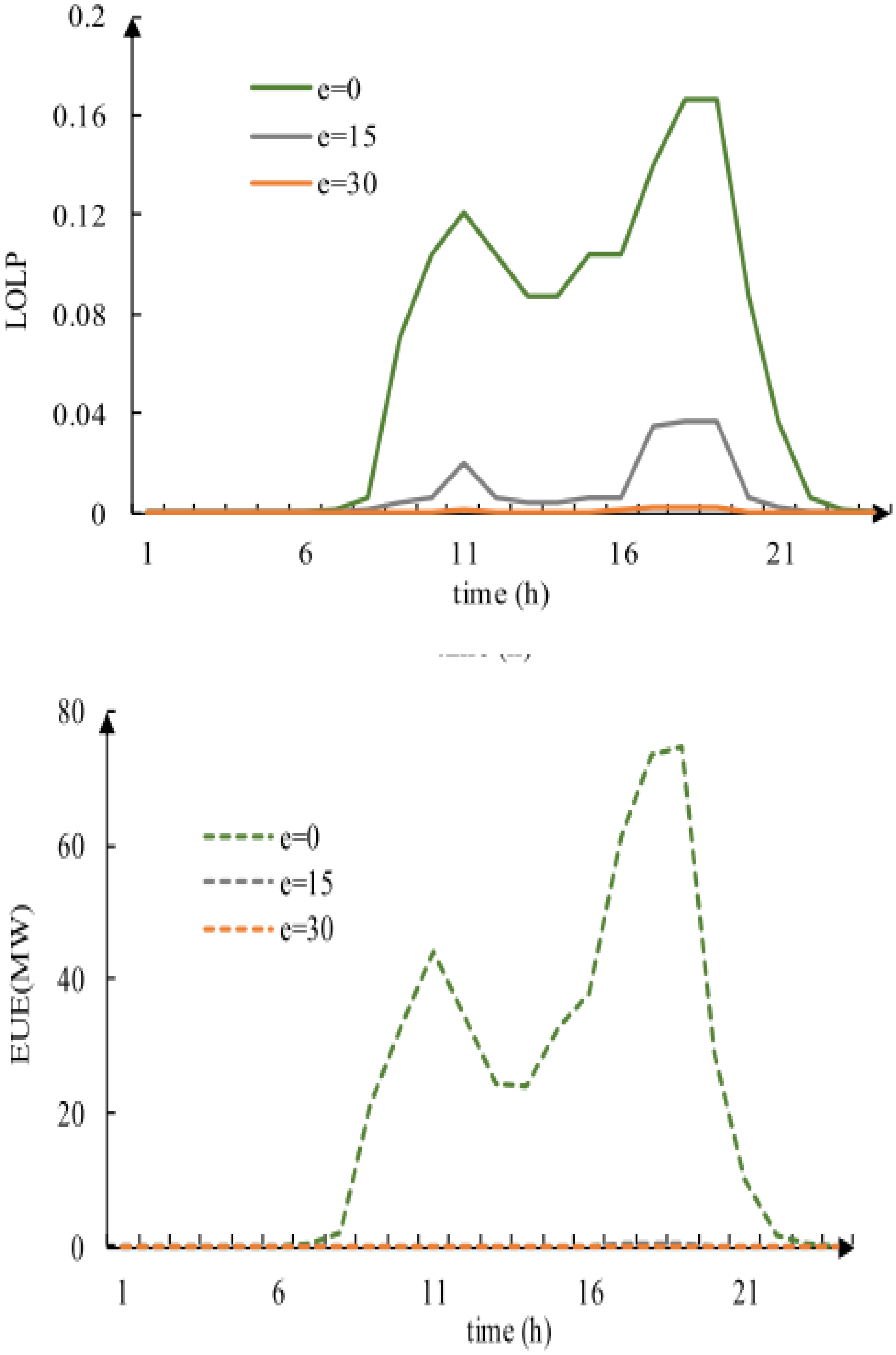


Fig. 6. LOLP and EUE under different elasticity coefficients.

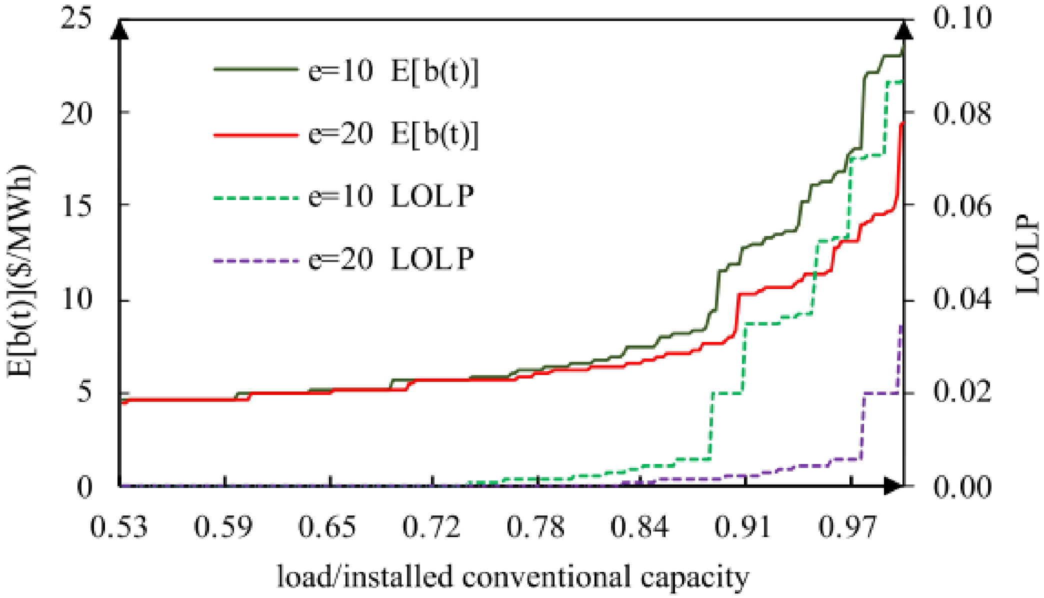
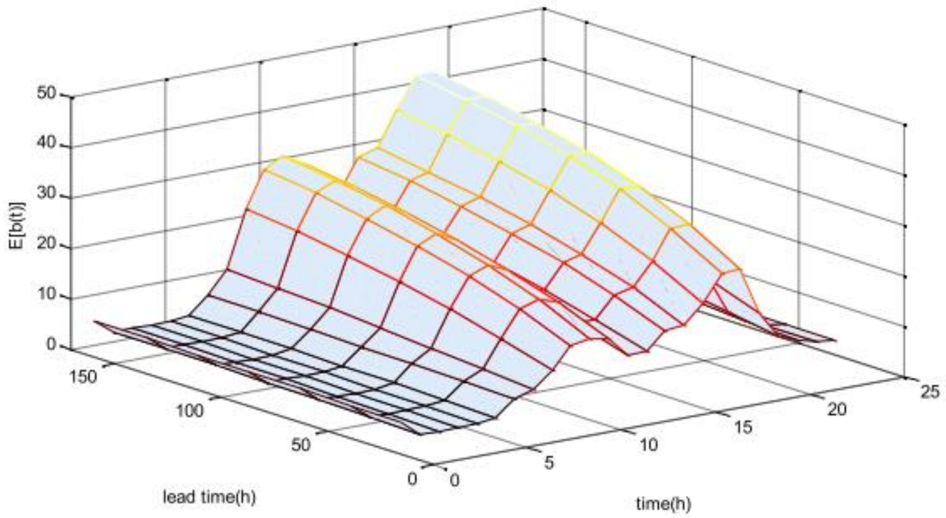


Fig. 7. The mean of marginal price and LOLP against different loads.

installed conventional capacity is 4.75GW, the ratio between load and installed conventional capacity ranges from 0.53 to 1. It can be noted that with the increase in load, marginal price, and LOLP are both increased. When the load-installed conventional capacity ratio is less than 0.88, the marginal price changes at a slow ascendant trend with the increase in load. When the ratio is higher than 0.88, the marginal price increases sharply. Compared with the price-elasticity coefficient of 10MW($/MWh)$^{-1}$, the sharp rise of marginal price and LOLP appeared later when the price-elasticity coefficient is 20 MW($/MWh)$^{-1}$. Consequently, in the base and mid-load periods, the marginal price increases in a slow ascendant trend with a higher load. In peak load periods, a price spike would happen with the increase in load. However, demand price elasticity could mitigate price spikes.

Effect of the availability of conventional generators on marginal price

To assess the influence of the conventional generator availability on marginal price, a sensitivity analysis is conducted with different lead time. A lower outage replacement rate of conventional generating units attributes to a shorter leading time. Fig. 8 shows the mean and variance of marginal price with the leading time of 24h, 48h, 72h, 96h, 120h, 144h, and 168h. It can be seen from Fig. 8 that with the decrease in leading time, the mean and variance of marginal price in 24h are both decreased. The reduction of the marginal price is 19.97 $/MWh at 19:00 with the leading time decreasing from 168 h to 24 h. The reduction of the marginal price is 0.97$/MWh at 4:00 with the leading time



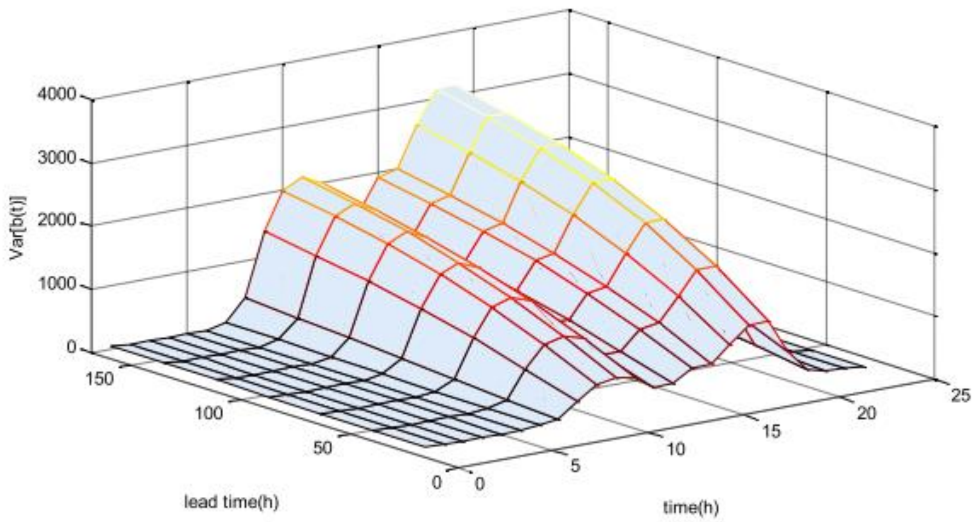


Fig. 8. Mean and variance of marginal price under different lead time.

decreasing from 168 h to 24 h. That is, with a shorter lead time, random outages of generation units can be mitigated, and the marginal price and its variance can also be reduced. In addition, in peak load periods, the availability of conventional generators has more significant influence on the marginal price.

Effect of wind power integration on marginal price and generation capacity adequacy

To verify the effect of wind power integration on marginal price and generation capacity adequacy, numerical simulations with different installed wind capacities, wind power uncertainties, and wind speeds are conducted. As wind turbine generators are assumed the same, we use different SDs of wind speeds to simulate diverse uncertainty levels of wind outputs, i.e., a larger SD of wind speed leads to a more significant uncertainty of wind outputs. Fig. 9, Fig. 10, and Fig. 11 demonstrate the impact of wind power integration on marginal price and generation capacity adequacy respectively, with load of 4.5 GW and price-elasticity coefficient of 10 MW($/MWh)$^{-1}$.

The impact of SDs of wind speed and installed wind capacity on marginal price and generation capacity adequacy indices are shown in Fig. 9 and Fig. 10. Five SD values ranging from 1 to 5 are tested. The installed capacity of wind farm ranges from 0.1 GW to 0.8GW (i.e., the ratio of installed wind over conventional capacity ranges from 0.021 to 0.168), and the average wind speed is set as 10 m/s. It can be seen that both the marginal price and LOLP decrease with a higher wind power wind capacity integration. Hence, the increase in wind power integration will reduce the marginal price and enhance the capacity adequacy. Meanwhile, both the marginal price and LOLP increase with wind speed of a higher SD.

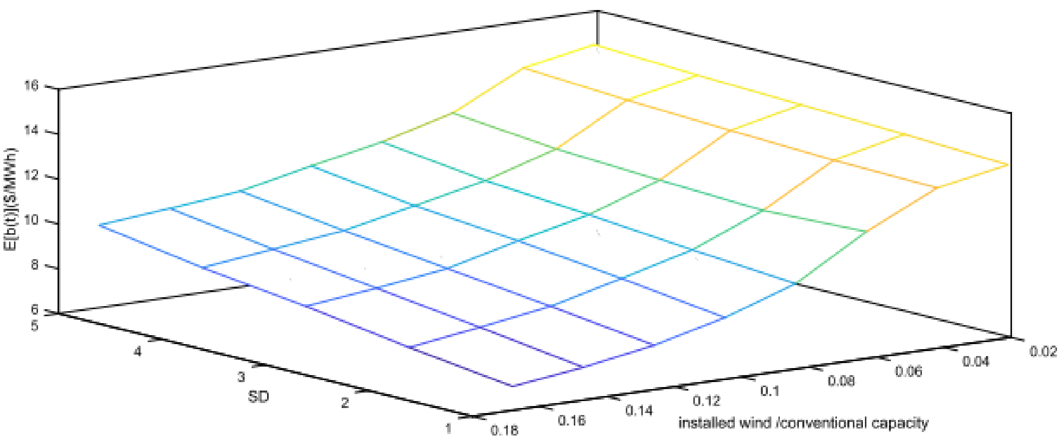


Fig. 9. Marginal price under different SDs of wind speed and installed

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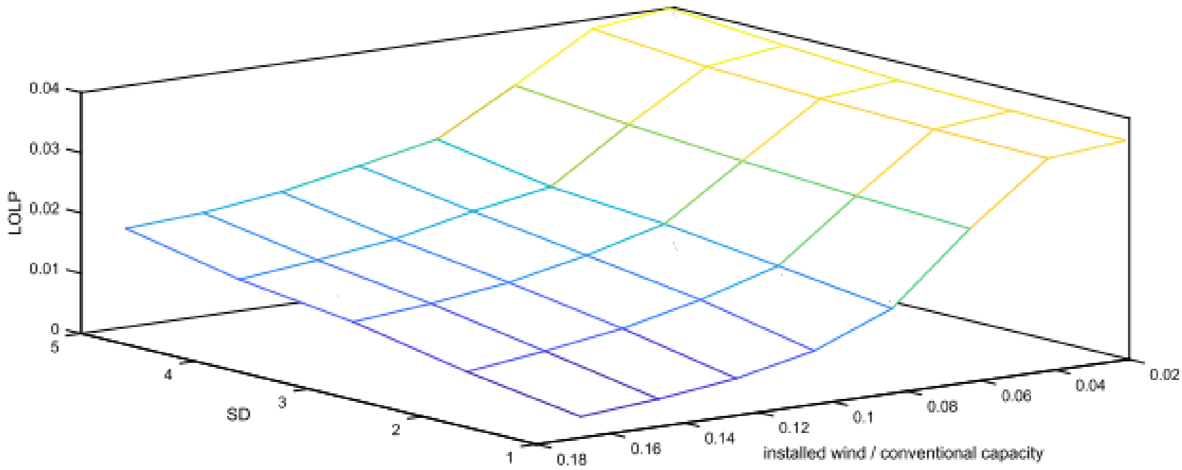

Fig. 10. LOLP under different SDs of wind speed and installed wind capacity

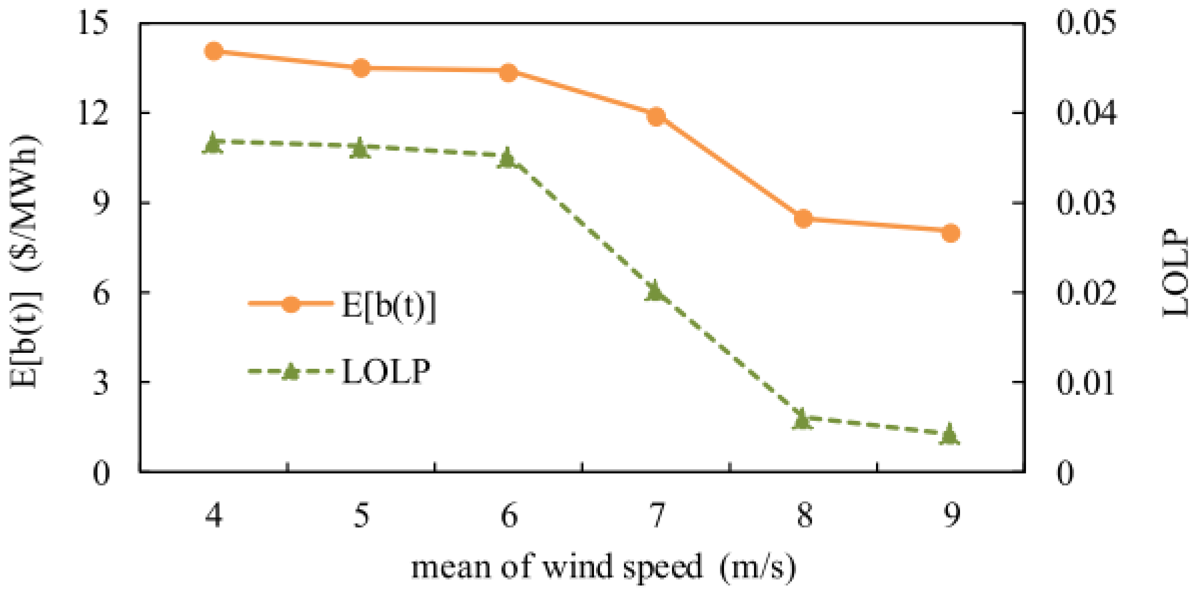

Fig. 11. Marginal price and LOLP under different mean value of wind speed.

Consequently, a more significant uncertainty of wind output leads to the increase in marginal price and may compromise capacity adequacy. Fig. 11 shows the marginal price and LOLP under different mean values of wind speed, with an installed wind capacity of 0.7GW. It can be observed that both the marginal price and LOLP decrease with a higher mean of wind speed. This clearly shows that the reduction in marginal price caused by wind power integration is more significant in higher wind speed periods.

Effect of generator's strategic bidding on marginal price and generation capacity adequacy

In electricity markets, producers may tend to exercise the market power to maximize their profits through strategic bidding. To verify the effect of generator's strategic bidding on marginal price and generation capacity adequacy, numerical simulations with different bidding strategies are conducted. Generators tend to exercise market power through increase their bidding price, especially in peak load periods. The bidding prices of the second bidding segments offered by each generator are assumed *W* times of that in Table I. Thirteen *W* values ranging from 1 to 2.2 are tested. Fig. 12 demonstrates the impact of generator's strategic bidding on marginal price and generation capacity adequacy respectively, with a peak load of 4.19 GW and price-elasticity coefficient of 0, 20 MW($/MWh)$^{-1}$. It can be noted that with a higher bidding price, the marginal price in each hour increase. Besides, the increase in marginal price is less significant for large demand-price elasticity. With a higher bidding price, LOLP in each hour decrease when the price-elasticity coefficient is 20

MW($/MWh)$^{-1}$, LOLP remains unchanged when the price-elasticity coefficient is 0 MW($/MWh)$^{-1}$. These results also demonstrate that elastic demand is helpful to mitigate generators' market power and improve the generation capacity adequacy.

CONCLUSION

Accurate electricity price forecasting and generation capacity adequacy evaluation need to adequately consider uncertainties of demand and supply sides, as well as impacts of demand-price elasticity. In this paper, a probabilistic method is developed to estimate marginal price and capacity adequacy of power systems with elastic demand and uncertainties of generation and demand sides. In this model, random outages of generating units with multi-bidding-segments, uncertainty of wind power outputs, along with demand-price elasticity are considered simultaneously. Numerical examples are presented to validate effectiveness of the proposed model. It is shown that: (i) with a high demand price elasticity, the marginal price and its variance can be reduced, and the capacity adequacy can also be effectively improved. Indeed, the influence is more significant during heavier load periods; (ii) in the base and mid-load periods, marginal price increases in a slow ascendant trend with a higher load. In peak load periods, a price spike would happen with the increase in loads. However, demand-price elasticity could mitigate price spikes; (iii) although the integration of wind power could decrease the marginal price and improve the capacity adequacy, with a significant wind output uncertainty, the marginal price will be high, and the capacity adequacy could be compromised. (iv) elastic demand is also helpful to mitigate generators' market power.

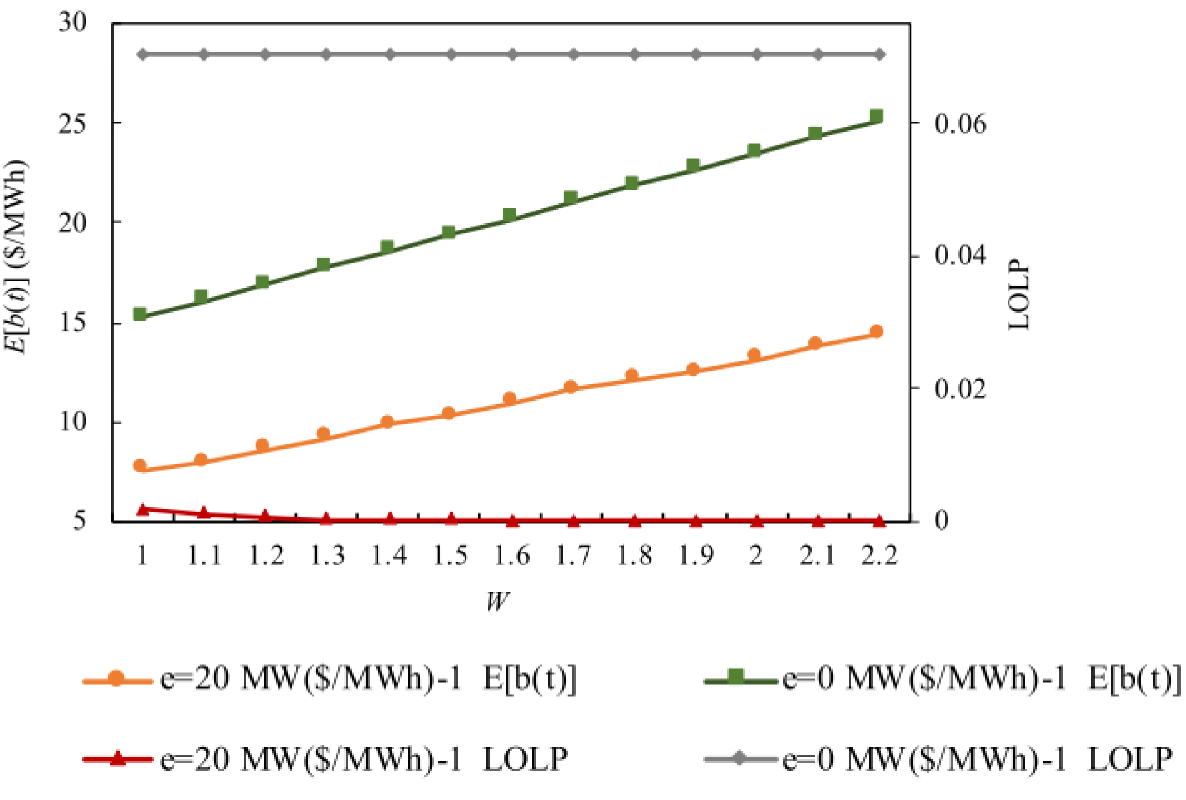

Fig. 12. Marginal price and LOLP under different bidding strategies of generators.

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